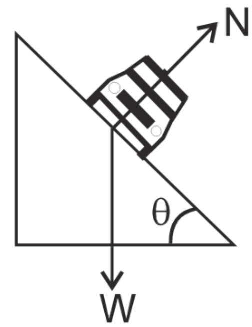
BANKING OF TRACKS

By Banking of Roads Only



Friction is not always reliable at circular turns if high speeds and sharp turns are involved to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction.

$$N\sin\theta = \frac{mv^2}{r}$$

$$N\cos\theta = mg$$

From these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \text{ or } v = \sqrt{rg\tan\theta}$$

Ex. What should be the angle of banking of a circular track of radius 600 m which is designed for cars at an average speed of 180 km/hr ?

Sol. Let the angle of banking be . The forces on the car are (figure)

(a) Weight of the car Mg downward and

(b) Normal force N.

For proper banking , static frictional force is not needed.

For vertical direction the acceleration is zero. So,

$$N\cos\theta = Mg \qquad \text{... (1)}$$

For horizontal direction, the acceleration is v^2/r towards the centre, so that

$$N\sin\theta = Mv^2/r \qquad \text{... (2)}$$

From(1) and(2), $\tan\theta = v^2/rg$

Putting the values, $\tan\theta = \frac{180(\text{ km/h})^2}{(600\text{ m})(10\text{ ms}^2)} = 0.4167 \Rightarrow \theta = 22.6°$

By Friction and Banking of Road Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.

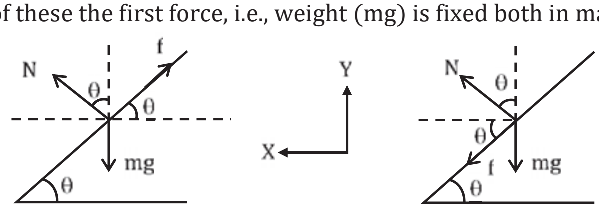


Figure (i) Figure (ii)

The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied up to a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m, and r, are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that:

1. Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component of weight mg sin is balanced by f.

2. Friction f will be inwards if

$$v > \sqrt{rg\tan\theta}$$

3. Friction f will be outwards if

$$v < \sqrt{rg\tan\theta}$$

4. Friction f will be zero if

$$v = \sqrt{rg\tan\theta}$$

5. For maximum safe speed (figure (ii)

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \text{... (1)}$$

$$N\cos\theta - f\sin\theta = mg \quad \text{... (2)}$$

As maximum value of friction f = mN

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$v_{mx} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 + \mu atn\theta)}}$$

The expression $\tan\theta = \frac{v^2}{rg}$also gives the angle of banking for an aircraft, i.e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.

The expression $\tan\theta = \frac{v^2}{rg}$ so gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, q is the angle which the cyclist must make with the vertical which will be discussed in chapter rotation.