STATIC FRICTION AND FRICTION GRAPH

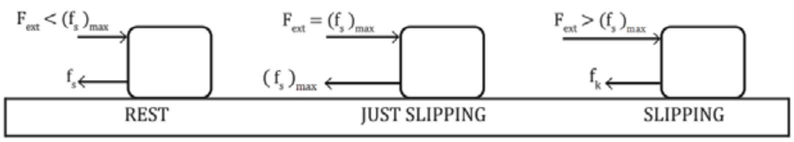
Static Friction

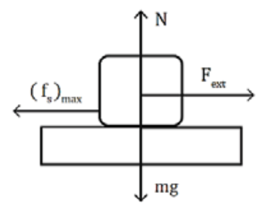
- When exerting force on an object at rest on a surface, movement initiates only when the applied force exceeds a specific threshold. Thus, a resisting force emerges, countering the tendency for motion, termed static friction.
- Static friction occurs when an individual attempts to move a stationary object on a surface, yet does not induce any relative motion between the body and the surface it rests upon.
- Static friction is a fluctuating resistance that matches and counters the external force until it exceeds the threshold for movement, initiating slipping. Therefore, static friction demonstrates self-regulating properties.

Limiting Friction

Limiting friction refers to the maximum frictional force achievable between two surfaces prior to the onset of sliding.

If the external force applied surpasses the magnitude of the limiting friction $\{(f_s)_{max}\}$ subsequently, slipping initiates and the type of friction transitions to kinetic friction.





The intensity of limiting friction correlates with the normal reaction exerted on the contact surface.

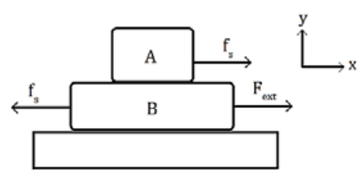
Limiting Friction ∝ Normal Reaction

$$(f_s)_{max} \propto N$$

$$(f_s)_{max} = \mu_s N$$

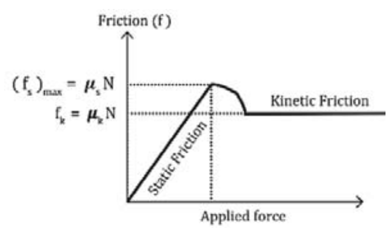
Where μ_s is the coefficient of static friction.

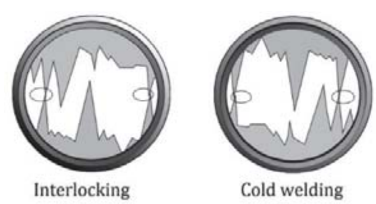
The direction of static friction opposes the inclination towards relative motion.



In the provided scenario, when an external force is applied, blocks A and B move collectively in the positive x-direction. However, the tendency of motion of block A concerning block B is in the negative x-direction. Consequently, the direction of static friction acting on block A will be in the positive x-direction.

Friction Graph





- The friction graph indicates that the limiting friction force exceeds the kinetic friction force. What is the reason behind this?
- During the prior session, we discovered that friction at a microscopic level results from interlocking and cold welding.
- When an object is stationary, it experiences both interlocking and cold welding, which jointly contribute to static friction. However, when there's relative motion between the surfaces, the bonds from cold welding break, leaving only interlocking to contribute to kinetic friction. Consequently, kinetic friction is lower than limiting friction.

$$(f_s)_{max} > f_k$$

$$\mu_s N > \mu_k N$$

$$\mu_s > \mu_k$$

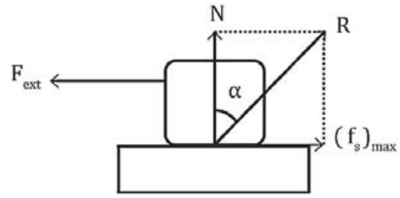
Hence, initiating relative motion between contacting surfaces is more challenging than sustaining it.

Note: Normally, $(f_s)_{max} = \mu_s N$ and, $f_k = \mu_k N$.

However, in a question if μ_s and μ_k Are not given separately, we can assume $\mu_s = \mu_k = \mu_{given}$

Angle of Friction (α)

It is the angle formed by the resultant of the normal reaction and limiting friction in relation to the normal reaction.

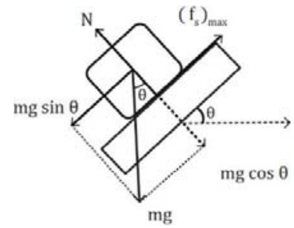


$$\tan\alpha = \frac{(f_s)_{max}}{N}$$
$$= \frac{\mu_s N}{N} = \mu_s$$

Angle of friction, $\alpha = \tan^{-1}(\mu_s)$

Angle of Repose (θ)

It is the smallest angle (θ) formed by an inclined plane with the horizontal at which an object positioned on the incline starts to slide.



$$N = mg\cos\theta$$
$$(f_s)_{max} = \mu_s N = \mu_s\, mg\cos\theta$$

When the object just start to slide,

$$(f_s)_{max} = mg\sin\theta$$
$$\mu_s\, mg\cos\theta = mg\sin\theta$$
$$\tan\theta = \mu_s$$

Angle of Repose, $\theta = \tan^{-1}(\mu_s)$

Ex. Consider a block kept on a horizontal surface as shown in the figure. For the given values of F, find the acceleration of the block, force of friction, and nature of friction in each case.

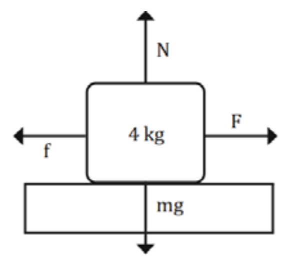
(Given, $\mu_s = 0.5, \mu_k = 0.2$ and $g = 10ms^{-2}$)

(a) F = 10 N (b) F = 15 N

(c) F = 20 N (d) F = 24 N

Sol. $N = mg = 4 \times 10 = 40N$

$(f_s)_{max} = \mu_s N = 0.5 \times 40 = 20N$



(a) When $F = 10$ N

$F < (f_s)_{max} \Rightarrow$ No Motion $\Rightarrow$ Static friction

Friction force, $f = F = 10$ N

Acceleration, $a = 0ms^{-2}$

(b) When $F = 15$ N

$F < (f_s)_{max} \Rightarrow$ No Motion $\Rightarrow$ Static friction

Friction force, $f = F = 15N$

Acceleration, $a = 0ms^{-2}$

(c) When $F = 20$ N

$F = (f_s)_{max} \Rightarrow$ Just start to move $\Rightarrow$ Limiting friction

Friction force, $f = (f_s)_{max} = 20$ N

Acceleration, $a = 0ms^{-2}$

(d) When $F = 24N$

$F > (f_s)_{max} \Rightarrow$ Slipping $\Rightarrow$ Kinetic friction

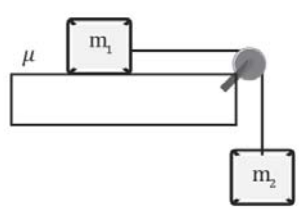
Friction force, $f = f_k = \mu_k N = 0.2 \times 40 = 8N$

Applying Newton's second law of motion,

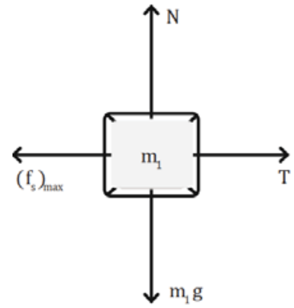
$$ma = F - f_k = 24 - 8 = 16N$$

Acceleration, $a = \frac{16}{m} = \frac{16}{4} = 4m^{-2}$

Ex. For the given arrangement, find the minimum value of mass m1 for which the system remains in equilibrium. Assume pulley and strings to be massless. $(\mu_k = \mu_s = \mu)$



Sol. As the system is under equilibrium, there is no relative motion. Hence static friction is acting on the mass m_1. Minimum value of m1 for equilibrium happens when maximum possible friction is offered by the surface on m_1. That is limiting friction acts between m1 and the surface.

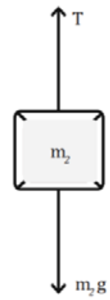


From the FBD of block m_1:

Balancing forces along the vertical and horizontal direction, we get,

$$N = m_1 g$$

$$T = (f_s)_{max} = \mu N = \mu m_1 g \quad \text{... (1)}$$



From the FBD of block m_2:

Balancing forces along the vertical direction, we get,

$$T = m_2 g$$

Equation (1) $\Rightarrow T = \mu m_1 g$

$$\mu m_1 g = m_2 g$$

If $m_1 < \frac{m_2}{\mu} \Rightarrow$ slipping occurs

If $m_1 \geq \frac{m_2}{\mu} \Rightarrow$ Equilibrium

Minimum value of m_1 for equilibrium $(m_1)_{min} = \frac{m_2}{\mu}$