

RELATIVE SLIPPING BETWEEN BLOCKS IN CONTACT

Ex. The top view of a block on a table is shown in the figure. Find the acceleration of the block. (Take $g = 10 \text{ ms}^{-2}$)

Sol. Total force acting on the block,

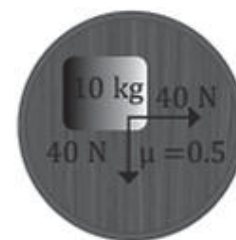
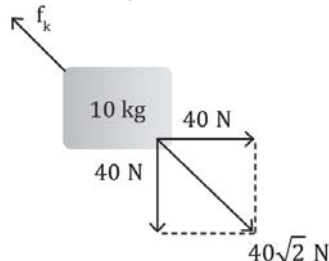
$$F = \sqrt{40^2 + 40^2} = 40\sqrt{2} \text{ N}$$

Kinetic friction,

$$f_k = \mu N = \mu mg = 0.5 \times 10 \times 10 = 50 \text{ N}$$

$$ma = F - f_k$$

$$a = \frac{F - f_k}{m} = \frac{40\sqrt{2} - 50}{10} = (4\sqrt{2} - 5) \text{ ms}^{-2}$$



Ex. A block of mass m rests on a horizontal floor. The coefficient of the static friction between the block and the floor is μ . It is desired to make the body move by applying the minimum possible force F . Find the magnitude of F and the direction in which it has to be applied.

Sol. Let F be the force applied on the block at an angle with the horizontal. θ When the body just starts to move, limiting friction is acting on the body. Equating forces along the vertical direction,

$$mg = N + F \sin \theta$$

$$N = mg - F \sin \theta$$

Limiting friction, $(f_s)_{\max} = \mu N = \mu(mg - F \sin \theta)$

Equating forces along the horizontal direction,

$$F \cos \theta = (f_s)_{\max}$$

$$= \mu(mg - F \sin \theta)$$

$$F \cos \theta + \mu F \sin \theta = \mu mg$$

$$F = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$

F is minimum when $\cos \theta + \mu \sin \theta$ is maximum.

Maximum value of $(\cos \theta + \mu \sin \theta) = \sqrt{1 + \mu^2}$

(\because Maximum value of $(a \cos \theta + b \sin \theta) = \sqrt{a^2 + b^2}$)

$$\text{Minimum value of } F = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

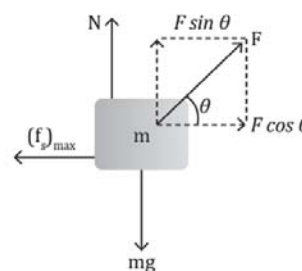
To find corresponding to maximum value of $\cos \theta + \mu \sin \theta$ differentiate, $\cos \theta + \mu \sin \theta$ with respect to θ and equate to zero.

$$\frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = 0$$

$$-\sin \theta + \mu \cos \theta = 0$$

$$\text{or } \tan \theta = \mu$$

$$\text{or } \theta = \tan^{-1}(\mu)$$



Ex. The friction coefficient between the box and the floor is μ . Find the maximum force that Jack can exert on the rope so that the box does not slip on the floor. (Given M is the mass of Jack, and m is the mass of the block.)

(a) $\frac{\mu(M+m)g}{1+\mu}$

(b) $\frac{\mu(M-m)g}{1-\mu}$

(c) $\frac{\mu(M+2m)g}{1+\sqrt{\mu}}$

(d) $\frac{\mu(M+m)g}{\sqrt{1+\mu}}$



Sol. From the FBD, it is clear that the box will not slide if the tension is less than the limiting friction. So, the maximum tension that can be applied without slipping is the limiting friction. Equating forces along the vertical direction,

$$(M + m)g = T + N$$

$$N = (M + m)g - T$$

Limiting friction, $(f_s)_{\max} = \mu N = \mu((M + m)g - T)$

Equating forces along the horizontal direction,

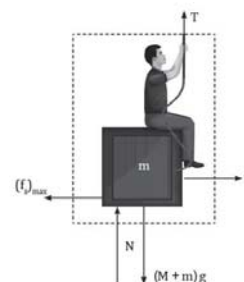
$$(f_s)_{\max} = T$$

$$\mu((M + m)g - T) = T$$

$$\mu(M + m)g = T(1 + \mu)$$

$$T = \frac{\mu(M+m)g}{1+\mu}$$

Hence, option (A) is the correct option.



Ex. Two blocks A and B of mass 10 kg each are arranged on each other as shown in the figure. The system is initially at rest. Find the acceleration of both the blocks.

Sol. Assumption: Let us assume that the blocks A and B move with a common acceleration a_c .

As the floor is smooth, there is no friction between the floor and block B.

Applying Newton's second law of motion, we get

$$F = (m_A + m_B)a_c$$

$$a_c = \frac{F}{m_A + m_B} = \frac{50}{10+10} = 2.5 \text{ ms}^{-2}$$

From the FBD of block A :

Equating forces along the y-direction,

$$N_A = m_A g = 100 \text{ N}$$

Along the x-direction,

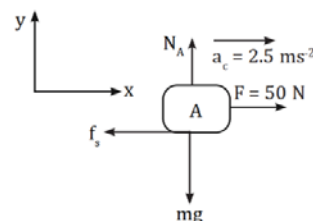
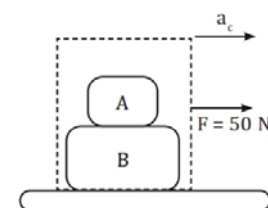
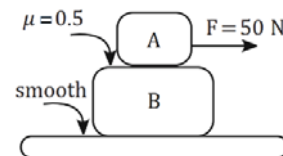
$$F - f_s = m_A a_c$$

$$f_s = F - m_A a_c = 50 - (10 \times 2.5) = 25 \text{ N}$$

$$(f_s)_{\max} = \mu N_A = 0.5 \times 100 = 50 \text{ N}$$

$$f_s < (f_s)_{\max} \Rightarrow \text{Assumption is correct.}$$

Hence, the blocks A and B move together with a common acceleration of 2.5 ms^{-2} .



Ex. Two blocks A and B of mass 10 kg each are arranged on each other as shown in the figure. The system is initially at rest. Find the acceleration of both the blocks.

Sol. Assumption: Let us assume that the blocks A and B move with a common acceleration a_c .

As the floor is smooth, there is no friction between the floor and block B.

Applying Newton's second law of motion, we get,

$$F = (m_A + m_B)a_c$$

$$a_c = \frac{F}{m_A + m_B} = \frac{110}{10+10} = 5.5 \text{ ms}^{-2}$$

From the FBD of block A based on the assumption:

Equating forces along the vertical direction,

$$N_A = m_A g = 100 \text{ N}$$

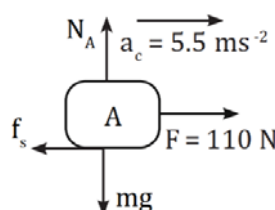
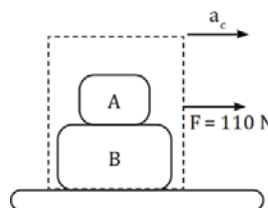
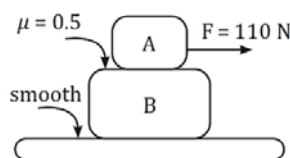
Along the horizontal direction,

$$F - f_s = m_A a_c$$

$$f_s = F - m_A a_c = 110 - (10 \times 5.5) = 55 \text{ N}$$

$$(f_s)_{\max} = \mu N_A = 0.5 \times 100 = 50 \text{ N}$$

The value of static friction obtained is greater than the maximum value of static friction. Hence, the assumption is wrong. So, A slips on B and kinetic friction is acting between A and B.



From the actual FBD of A,

Along y direction, $N_A = m_A g = 100 \text{ N}$

Kinetic friction, $f_k = \mu N_A = 0.5 \times 100 = 50 \text{ N}$

Along x direction, $F - f_k = m_A a_A$ or $110 - 50 = m_A a_A$

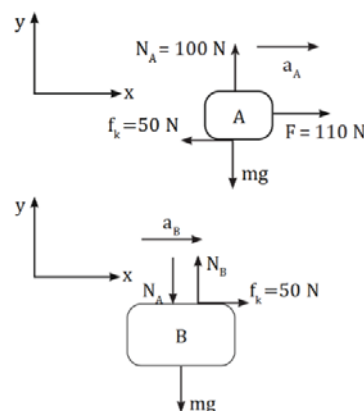
$$a_A = \frac{60}{10} = 6 \text{ ms}^{-2}$$

From the FBD of B

From the actual FBD of A, On block B, kinetic friction acts along the positive x-direction and causes the block to accelerate in the positive x-direction.

$$f_k = m_B a_B = 50 \text{ N}$$

$$\text{Acceleration, } a_B = \frac{50}{m_B} = \frac{50}{10} = 5 \text{ ms}^{-2}$$



Ex. A 2 kg block is placed over a 4 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0.20. Find the acceleration of the two blocks if a horizontal force of 12 N is applied on the upper block. (Take $g = 10 \text{ ms}^{-2}$).

Sol. Assumption: Let us assume that both the blocks move together with a common acceleration a_c .

As the floor is smooth, there is no friction between the floor and the 4 kg block.

Applying Newton's second law of motion, we get,

$$F = (m_A + m_B) a_c$$

$$a_c = \frac{F}{m+m} = \frac{12}{2+4} = 2 \text{ ms}^{-2}$$

From the FBD of the 2 kg block based on the assumption,

Along y direction, $N_1 = mg = 2 \times 10 = 20 \text{ N}$

Along x direction, $F - f_s = ma_c$

$$f_s = F - ma_c$$

$$f_s = 12 - ma_c = 12 - (2 \times 2) = 8 \text{ N}$$

Limiting friction, $(f_s)_{\max} = \mu N_1 = 0.2 \times 20 = 4 \text{ N}$

So, the value of static friction obtained is more than the maximum value of static friction. Hence, the assumption is wrong. So, the 2 kg block slips on the 4 kg block and kinetic friction is acting between the blocks.

From the actual FBD of the 2 Kg block

Along y direction, $N_1 = mg = 2 \times 10 = 20 \text{ N}$

Kinetic friction, $f_k = \mu N = 0.2 \times 20 = 4 \text{ N}$

Along x direction, $F - f_k = m_A a_A = \frac{F - f_k}{m} = \frac{12 - 4}{2} = 4 \text{ ms}^{-2}$

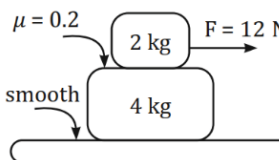
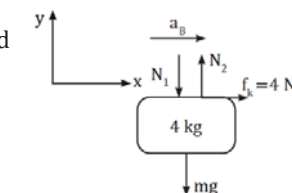
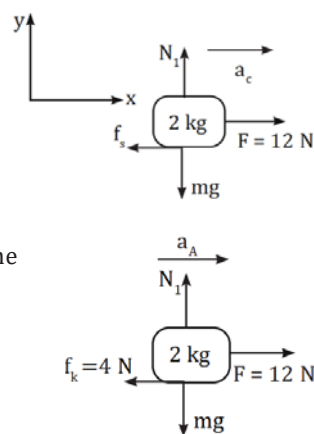
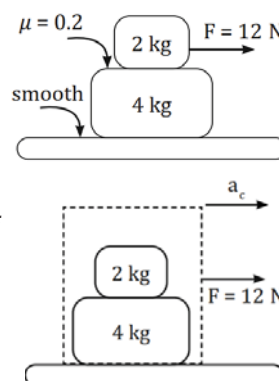
From the FBD of 4 kg block,

On the 4 kg block, kinetic friction acts along the positive x-direction and causes the block to accelerate in the positive x-direction.

Applying Newton's second law of motion, we get,

$$f_k = m_B a_B = 4 \text{ N}$$

$$\text{Acceleration, } a_B = \frac{4}{4} = 1 \text{ ms}^{-2}$$



Ex. A 2 kg block is placed over a 4 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0.2. Find the acceleration of the two blocks if a horizontal force of 12 N is applied on the lower block. ($g = 10 \text{ ms}^{-2}$)

Sol. Assumption: Both the blocks move together with a common acceleration a_c .

Applying Newton's second law of motion, we get,

$$F = (m_A + m_B)a_c$$

$$a_c = \frac{F}{(m_A + m_B)} = \frac{12}{2+4} = 2\text{ m}^{-2}$$

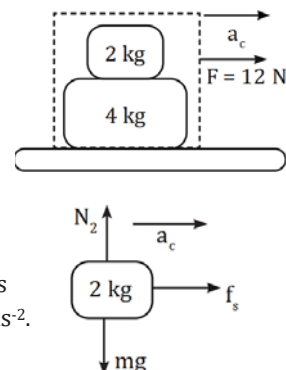
From the FBD of 2 Kg block,

$$N_2 = mg = 20\text{ N}$$

$$f_s = ma_c = 2 \times 2 = 4\text{ N}$$

$$(f_s)_{\max} = \mu N_2 = 0.2 \times 20 = 4\text{ N}$$

So, limiting friction acts between the blocks and hence, the assumption is correct. Thus, both the blocks move together with an acceleration of 2 ms^{-2} .



Guidelines for resolving block-on-block scenarios

1. Let's suppose all the blocks move collectively with a shared acceleration, denoted as a_c .
2. Determine the static friction force (f_s) necessary for the block to move with the common acceleration.
3. Find the limiting friction (f_s)_{max}.
4. If $f_s \leq (f_s)_{\max}$ Both will move simultaneously with a shared acceleration a_c .
5. If $f_s > (f_s)_{\max}$, If there is relative motion between them, then either the assumption is incorrect, or they will move with different accelerations. Determine the individual acceleration of each block from their respective free body diagrams.

Ex. A block of mass 1 kg is attached to the spring as shown in the figure. Find the range (R) in which the block can be kept without slipping.

- (A) 1 m (B) 1.5 m (C) 1.6 m (D) 1.8 m

Sol. If the mass M is moved, the spring undergoes deformation. As a result, spring force is developed that tries to restore the equilibrium position. However, if the spring force is less than the limiting friction force, then it fails to do so. So, we need to find the extension and contraction at limiting friction. The sum of these is the range in which the block can be kept without slipping.

From the FBD of 1 kg block when the spring is stretched,

Along the y-direction,

$$N = Mg \cos 37^\circ = 1 \times 10 \times \frac{4}{5} = 8\text{ N}$$

$$(f_s)_{\max} = \mu_s N = 1 \times 8 = 8\text{ N}$$

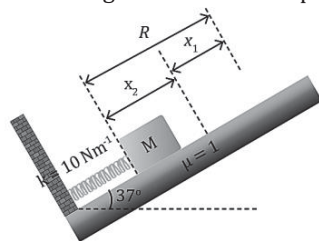
Along the x-direction,

$$kx_1 + Mg \sin 37^\circ = (f_s)_{\max}$$

$$x_1 = \frac{(f_s)_{\max} - Mg \sin 37^\circ}{k}$$

$$= \frac{8 - (10 \times \frac{3}{5})}{10} = 0.2\text{ m}$$

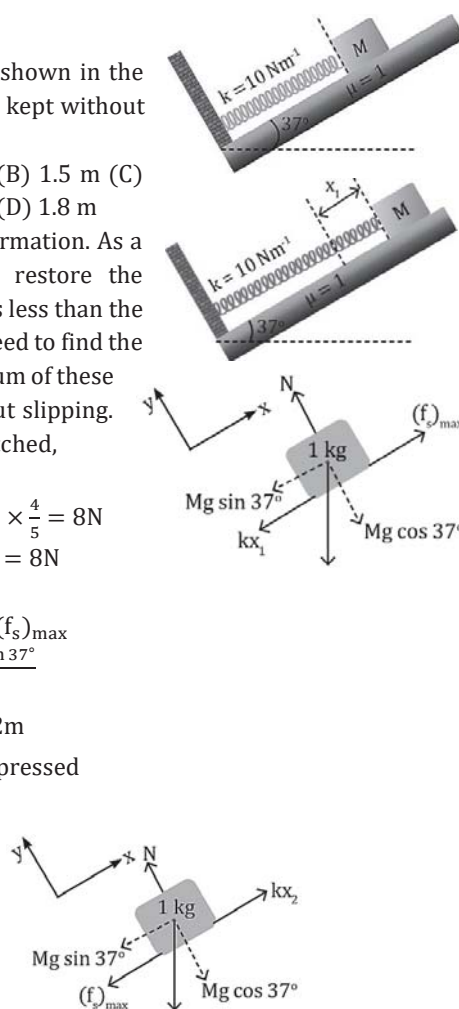
From the FBD of 1 kg block when the spring is compressed



Along the y-direction,

$$N = Mg \cos 37^\circ = 1 \times 10 \times \frac{4}{5} = 8\text{ N}$$

$$(f_s)_{\max} = \mu_s N = 1 \times 8 = 8\text{ N}$$



Along the x-direction,

$$\begin{aligned} Mg \sin 37^\circ + (f_s)_{\max} &= kx_2 \\ x_2 &= \frac{Mg \sin 37^\circ + (f_s)_{\max}}{k} \\ &= \frac{(10 \times \frac{3}{5}) + 8}{10} = 1.4 \text{ m} \end{aligned}$$

$$\text{Range} = x_1 + x_2 = 0.2 + 1.4 = 1.6 \text{ m}$$

Thus, option (C) is the correct answer.

Ex. Calculate the force F required to cause the block of weight $W_1 = 200 \text{ N}$ to just slide under the block of weight $W_2 = 100 \text{ N}$ as shown in the figure. The coefficient of friction for all the surfaces in contact is equal to 0.25. Find the tension in the rod.

Sol. As the block W_1 is just starting to slide, the friction force acting is the limiting friction.

From the FBD of W_2 block,

Along the y-direction,

$$\begin{aligned} N_2 + T \sin 45^\circ &= W_2 \\ N_2 + \frac{T}{\sqrt{2}} &= 100 \quad \dots (1) \end{aligned}$$

Along the x-direction,

$$\begin{aligned} T \cos 45^\circ &= f_2 = \mu N_2 = 0.25 N_2 \\ \frac{T}{\sqrt{2}} &= 0.25 N_2 \quad \dots (2) \end{aligned}$$

From equation (1), we get the following:

$$\begin{aligned} N_2 + \frac{T}{\sqrt{2}} &= 100 \\ N_2 + 0.25 N_2 &= 100 \\ N_2 &= 80 \text{ N} \end{aligned}$$

Substituting the value of N_2 in equation (2), we get the following:

$$\begin{aligned} \frac{T}{\sqrt{2}} &= 0.25 N_2 = 0.25 \times 80 = 20 \\ T &= 20\sqrt{2} \text{ N} \end{aligned}$$

Hence, the tension in the rod is $20\sqrt{2} \text{ N}$.

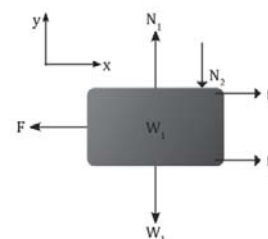
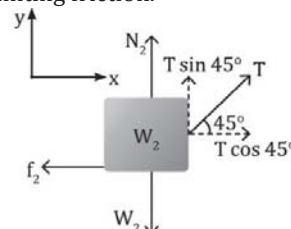
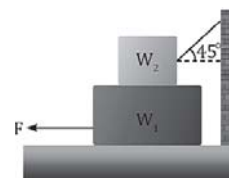
From the FBD of W_1 block.

Along the y-direction,

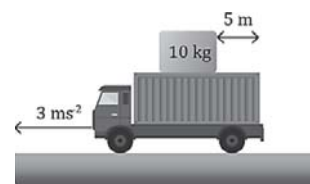
$$\begin{aligned} N_1 &= N_2 + W_1 \\ N_1 &= 80 + 200 = 280 \text{ N} \end{aligned}$$

Along the x-direction,

$$\begin{aligned} F &= f_1 + f_2 \\ &= \mu N_1 + \mu N_2 \\ &= (0.25 \times 280) + (0.25 \times 80) = 90 \text{ N} \end{aligned}$$



Ex. A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in the figure. The coefficient of friction between the block and the trolley is 0.2. The trolley is given a uniform acceleration of 3 ms^{-2} towards the left, starting from rest. Find the distance travelled by the trolley from the starting point when the block falls off.



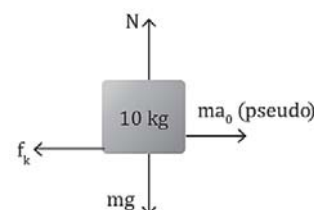
Sol. FBD of 10 kg block taking the trolley as frame of reference is as shown in the figure. The trolley moves in the leftward direction with an acceleration of a_0 . Hence, the pseudo force on the 10 kg block acts in the rightward direction that causes the block to move in the rightward direction with respect to the trolley.

Pseudo force = mass of body \times acceleration of frame of reference

$$\begin{aligned} &= 10 \times a_0 \\ &= 10 \times 3 = 30 \text{ N} \end{aligned}$$

Along the y-direction, $N = mg = 10 \times 10 = 100 \text{ N}$

Limiting friction, $(f_s)_{\max} = \mu N = 0.2 \times 100 = 20 \text{ N}$



Hence, the body will move in the rightward direction with an acceleration and kinetic friction acting on the body.

Let 'a' be the acceleration of the block taking the trolley as the frame of reference.

$$\begin{aligned}\text{Along the x-direction,} \quad F_{\text{pseudo}} - f_k &= ma \\ ma_0 - f_k &= ma \\ 10 \times 3 - \mu N &= 10a \\ 30 - (0.2 \times 100) &= 10a \\ a &= 1 \text{ ms}^{-2}\end{aligned}$$

Now, in the trolley frame, the block undergoes a uniformly accelerated motion and covers a displacement of 5 m before falling. Let 't' be the time taken by the block to cover 5 m. Using the second equation of motion, we get,

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\ 5 &= 0 + \frac{1}{2} \times 1 \times t^2 \\ t &= \sqrt{10} \text{ s}\end{aligned}$$

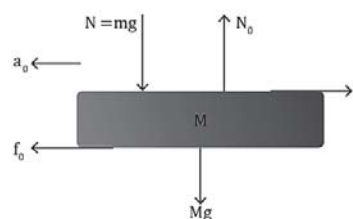
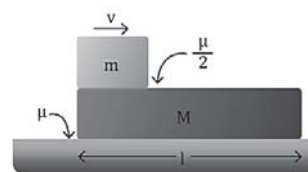
In order to find the distance travelled by the trolley in time t, taking ground as the frame of reference. Using the second equation of motion, we get

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\ &= 0 + \frac{1}{2} \times 3 \times 10 (\because t = \sqrt{10} \text{ s}) = 15 \text{ m}\end{aligned}$$

Ex. A block of mass m is placed on another block of mass M and length l as shown in the figure. If the system starts moving with a velocity v towards the right, then find the time elapsed before the smaller block separates from the bigger block.

Sol. Initially, both the blocks of masses m and M have a velocity of v. If there were no friction between M and ground, then m and M will continue to move together with constant velocity v. However, due to the friction between M and ground, M block will slow down. Or we can say that the M block has an acceleration 'a₀' in the leftward direction with respect to the ground frame.

When the M block slows down, the m block will slide in the right direction with respect to block M. So, by taking M as the frame of reference, m has an acceleration in the rightward direction. So, the friction on m will act in the leftward direction. From the FBD of M,



$$N_0 = mg + Mg = (M + m)g$$

Let f_0 be friction on M exerted by ground and f be the friction on M exerted by m.

$$\begin{aligned}f &= \frac{\mu}{2} N = \frac{\mu mg}{2} \\ f_0 &= \mu N_0 = \mu(M + m)g\end{aligned}$$

Along the horizontal direction,

$$\begin{aligned}f_0 - f &= Ma_0 \\ \mu N_0 - \frac{\mu}{2} N &= Ma_0 \\ \mu(M + m)g - \frac{\mu}{2} mg &= Ma_0 \\ \mu g \left(\frac{2M + m}{2} \right) &= Ma_0 \\ a_0 &= \frac{\mu(2M + m)g}{2M}\end{aligned}$$

From the FBD of m taking M as frame of reference,

$$\begin{aligned}\text{Along the horizontal direction,} \quad ma_0 - f &= ma \\ \left(m \times \frac{\mu(2M + m)g}{2M} \right) - \left(\frac{\mu}{2} \times mg \right) &= ma\end{aligned}$$

$$a = \frac{\mu g}{2} \left(\frac{2M+m}{M} - 1 \right)$$

$$a = \frac{\mu g}{2} \left(\frac{M+m}{M} \right)$$

So, the m block will slide on M block with an acceleration a and covers a distance of l before the fall. In order to find out the time taken, using the second equation of motion, we get,

$$s = ut + \frac{1}{2}at^2$$

$$l = 0 + \frac{1}{2} \left(\frac{\mu(M+m)g}{2M} \right) t^2$$

(Since, the initial velocity of the smaller block with respect to the larger block is zero.)

$$t^2 = \frac{4Ml}{\mu(M+m)g}$$

$$\text{Time taken before fall, } t = \sqrt{\frac{4Ml}{\mu(M+m)g}}$$

Ex. If a horizontal force of 10 N is applied on the 3 kg block, find the accelerations a_1 , a_2 , and a_3 of the three blocks shown in the figure. [Take $g = 10 \text{ ms}^{-2}$]

Sol. Assumption: Let us assume that all the three blocks move together with common acceleration of a_c .

$$a_c = \frac{10}{2+3+7} = \frac{10}{12} = \frac{5}{6} \text{ ms}^{-2}$$

From the FBD of 2 kg block,

$$N_1 = m_1 g = 2 \times 10 = 20 \text{ N}$$

$$f_s = m_1 \times a_c = 2 \times \frac{5}{6} = \frac{5}{3} \text{ N}$$

$$(f_s)_{\max} = \mu_1 N_1 = 0.2 \times 20 = 4 \text{ N}$$

$$f_s < (f_s)_{\max}$$

Hence, the 2 kg and 3 kg blocks will move together. So, the 2 kg and 3 kg blocks can be replaced with a single block of 5 kg.

From the FBD of 5 kg block,

$$N_2 = m_2 g = 5 \times 10 = 50 \text{ N}$$

Applying Newton's second law of motion, we get,

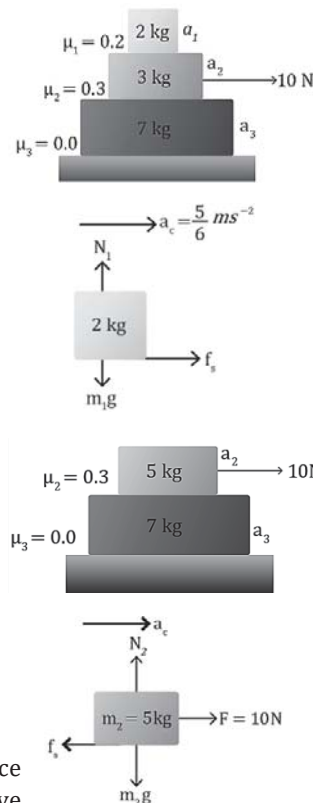
$$F - f_s = m_2 a_c$$

$$f_s = F - m_2 a_c = 10 - \left(5 \times \frac{5}{6} \right) = \frac{35}{6} \text{ N}$$

$$(f_s)_{\max} = \mu_2 N_2 = 0.3 \times 50 = 15 \text{ N}$$

$$f_s < (f_s)_{\max}$$

Hence, the 5 kg and 7 kg blocks will move together. Since the surface on which the blocks are kept is smooth, all the three blocks will move together with a common acceleration of $\frac{5}{6} \text{ ms}^{-2}$

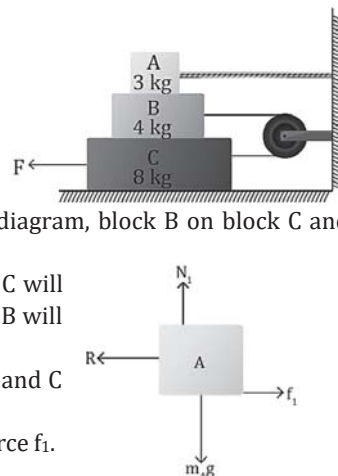


Ex. Blocks A, B, and C weigh 3 kg, 4 kg, and 8 kg, respectively. The coefficient of sliding friction between any two surfaces is 0.25. The block A is held at rest by a massless rigid rod fixed to the wall, while block B and block C are connected by a light flexible cord passing around a frictionless pulley. Find the force F necessary to drag block C along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, block B on block C and block A on block B, is maintained all through. ($g = 10 \text{ ms}^{-2}$)

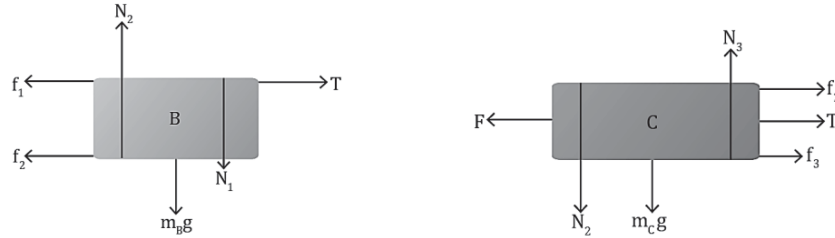
Sol. Block A will be at rest as it is connected to the rigid rod. Block C will move in the left direction as an effect of the applied force. Block B will move in the right direction.

Let f_1 , f_2 , and f_3 be the kinetic friction between A and B, B and C, and C and ground, respectively.

R is the normal reaction offered by the rigid rod against friction force f_1 .



From the FBD of block A, $N_1 = m_A g = 30\text{N}$
 $f_1 = \mu N_1$
 From the FBD of block B, $N_2 = N_1 + m_B g = 30 + 40 = 70\text{N}$
 $T = f_1 + f_2$
 From the FBD of block C, $N_3 = N_2 + m_C g = 70 + 80 = 150\text{N}$
 $F = T + f_2 + f_3$
 $= f_1 + 2f_2 + f_3 (\because T = f_1 + f_2)$
 $= \mu N_1 + 2\mu N_2 + \mu N_3$
 $= \mu(N_1 + 2N_2 + N_3)$
 $= 0.25(30 + 140 + 150) = 80\text{N}$



- Ex.** If a horizontal force of 10 N is applied on the 7 kg block, find the accelerations a_1 , a_2 , and a_3 of the three blocks shown in the figure.
Sol. Assumption: Let us assume that all the three blocks move together with common acceleration of a_c .

$$a_c = \frac{10}{2+3+7} = \frac{10}{12} = \frac{5}{6} \text{ ms}^{-2}$$

From the FBD of 2 kg block,
 $N_1 = m_1 g = 2 \times 10 = 20\text{N}$
 $f_s = m_1 \times a_c = 2 \times \frac{5}{6} = \frac{5}{3}\text{N}$
 $(f_s)_{\max} = \mu_1 N_1 = 0.2 \times 20 = 4\text{N}$
 $f_s < (f_s)_{\max}$

Hence, the 2 kg and 3 kg blocks will move together. So, the 2 kg and 3 kg blocks can be replaced with a single block of 5 kg.

From the FBD of 5 kg block,

$$N_2 = m_2 g = 5 \times 10 = 50\text{N}$$

$$f_s = m_2 a_c = 5 \times \frac{5}{6} = \frac{25}{6}\text{N}$$

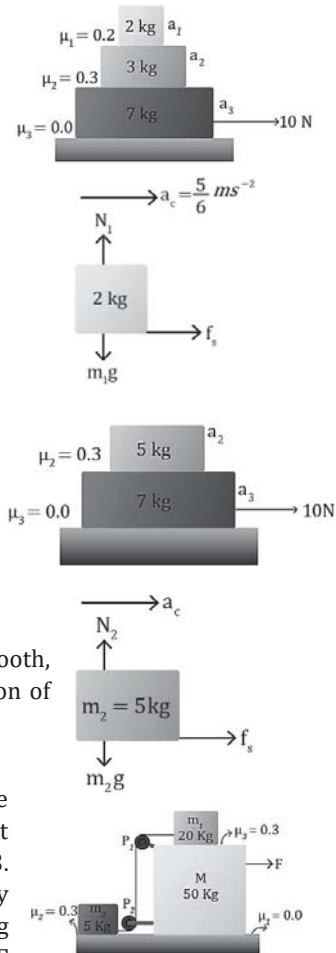
$$(f_s)_{\max} = \mu_2 N_2 = 0.3 \times 50 = 15\text{N}$$

$$f_s < (f_s)_{\max}$$

Hence, the 5 kg and 7 kg will move together. Since the surface is smooth, all the three blocks will move together with a common acceleration of $\frac{5}{6} \text{ ms}^{-2}$

- Ex.** Masses m_1 , m_2 , and M are 20 kg, 5 kg, and 50 kg, respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between M and m_1 , and that between m_2 and ground is 0.3. The pulleys and strings are massless. The string is perfectly horizontal between P_1 and m_1 , and also between P_2 and m_2 . The string is perfectly vertical between P_1 and P_2 . An external horizontal force F is applied to the mass M . (Take $g = 10 \text{ ms}^{-2}$)

- (a) Let the magnitude of force of friction between M and m_1 be f_1 , and between m_2 and ground be f_2 . For a particular F , it is found that $f_1 = 2f_2$. Find f_1 and f_2 . Write down the equations of all the masses, and find F , tension, and acceleration of all the masses.
 (b) Draw the FBD of mass M clearly showing the forces.



Sol. Let 'a' be the acceleration of m_2 in the right direction. From the FBD of block m_2 ,

$$N_2 = m_2 g$$

$$f_2 = \mu_2 N_2 = \mu_2 m_2 g = 0.3 \times 5 \times 10 = 15 \text{ N}$$

Along the horizontal direction,

$$T - 15 = 5a \quad \dots (1)$$

With respect to M, m_1 has a tendency to slip backward. So, the friction acts in a forward direction.

$$f_1 = 2f_2 \text{ (Given in the question)}$$

$$= 2 \times 15 = 30 \text{ N}$$

$$N_1 = m_1 \times g = 20 \times 10 = 200 \text{ N}$$

$$(f_{1s})_{\max} = \mu_3 \times N_1 = 0.3 \times 200 = 60 \text{ N}$$

$$f_1 < (f_{1s})_{\max}$$

So, the static friction is acting between m_1 and M. Hence, there is no slip between m_1 and M. In other words, they move together. So, the whole system will move rightward with common acceleration 'a'. Hence, in the horizontal direction,

$$f_1 - T = m_1 a$$

$$30 - T = 20a$$

From the FBD of M block,

In the horizontal direction,

$$F - f_1 = Ma$$

$$F - (T + m_1 a) = Ma (\because f_1 = T + m_1 a)$$

$$F - T - 20a = 50a$$

$$F - T = 70a$$

... (2)

Adding equations (1) and (2),

$$T - 15 + 30 - T = 25a$$

$$15 = 25a$$

$$a = \frac{15}{25} = \frac{3}{5} \text{ m}^{-2}$$

Substituting value of a in equation (1),

$$T - 15 = 5a = 5 \times \frac{3}{5} = 3$$

$$T = 3 + 15 = 18 \text{ N}$$

Substituting value of a and T in equation (2),

$$F - 18 = 70a = 70 \times \frac{3}{5} = 42$$

$$F = 60 \text{ N}$$

