WEIGHING MACHINE AND SPRING

Massless Body

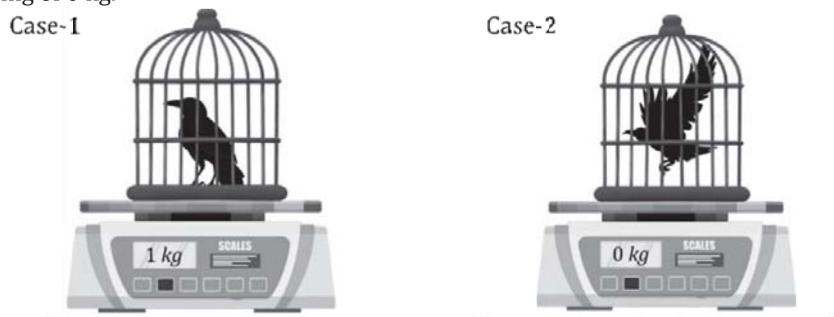
In the case of massless bodies ($m \to 0$), the net force $\sum\vec{F} = 0$; equals zero; nevertheless, there is a finite acceleration (a).

In a massless string, a consistent tension is maintained uniformly along its entire length.

'Light-weight' can serve as a synonym for 'massless' in certain contexts.

Weighing Machine

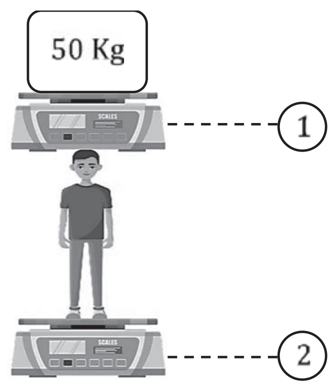
It is a device designed to gauge the applied normal force. To illustrate, consider a raven as an example. Suppose a weighing machine is adjusted so that when a cage is placed on it, it registers a reading of 0 kg.



Raven is sitting on a weighing machine and the reading is 1 kg. Raven is fluttering in the cage and the reading is 0 kg.

The disparities in readings arise due to the raven exerting its weight on the machine in the first case. In the second scenario, where the raven doesn't apply any normal force, the reading is zero. Normal reaction occurs only when two bodies are in contact. Similarly, if you place one leg on the weighing machine and the other on the ground, the machine will display approximately half of your weight as the reading. This demonstrates that the machine gauges the force with which you press against it. Therefore, the weighing machine measures the normal reaction it experiences, not the actual weight of the body.

Ex. A 60 kg man stands on a weighing machine with a mass of 5 kg, positioned on the ground. Another identical weighing machine is positioned above the man's head. A block weighing 50 kg is placed on the second weighing machine. Calculate the readings displayed by both weighing machines. (Where $g = 10\ ms^{-2}$)



Sol. Consider m_1 as the mass of weighing machine 1 and m_2 as the mass of weighing machine 2. Let m_{block} represent the mass of the block, and m_{man} denote the mass of the man.

Reading of weighing machine (1)

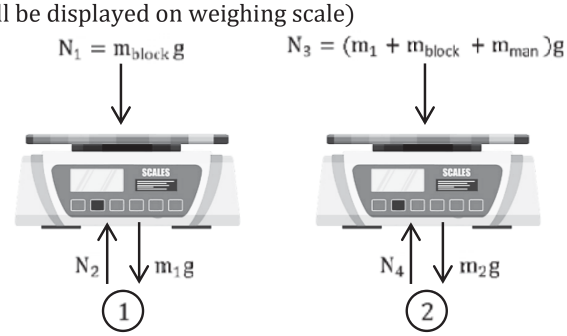
$$N_1 = (m_{block})g = 50g = 500\ N$$

(Or 50 kg, as will be displayed on weighing scale)

Reading of weighing machine (2)

$N_3 = (m_1 + m_{block} + m_{man})g = (5 + 50 + 60)g = 115g = 1150\ N$

(Or 115 kg, as will be displayed on weighing scale)



Ex. A man with a mass of 60 kg stands on a lightweight weighing machine placed inside a box weighing 30 kg. The box is suspended from a pulley attached to the ceiling by a light rope, with the man holding the other end. If the man successfully maintains the box in a state of rest, what reading does the weighing machine display? (Where $g = 10\ ms^{-2}$)

(A) 30 kg (B) 15 kg
(C) 45 kg (D) 60 kg

Sol. Since the string is massless, it will uniformly convey the same tension. Considering the Free Body Diagram (FBD) of the man, let N_1 represent the normal reaction applied by the machine on the man, and T denote the tension in the string.

$$N_1 + T = 600 \quad \text{... (1)}$$

Consider N as the normal reaction exerted on the box by the weighing machine.
The force remains unchanged as it passes through a massless body. Consequently, the normal force applied by the weighing machine on the box equals the normal reaction exerted on the weighing machine by the man.

$$N = N_1$$

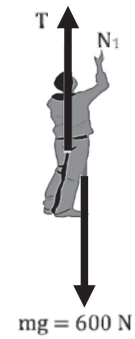
From the FBD of the box

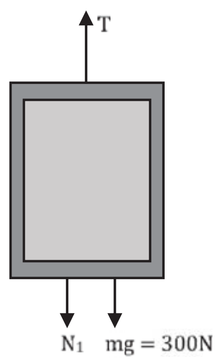
$$T - N_1 = 300 \quad \text{... (2)}$$

Subtracting equation (2) from equation (1)

$$2N_1 = 300\ N$$
$$N_1 = 150\ N$$

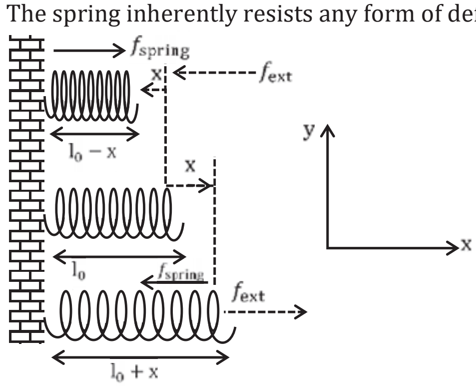
15 kg mass will be shown by the weighing machine.
Hence, option (B) is correct.





Ideal Spring.

An ideal spring is constructed using an ideal string (which is both massless and inextensible) wound in the shape of a helix.
The spring's length can change when force is applied, owing to its design.
Restorative Property: The spring inherently resists any form of deformation.



> Common feature of an ideal spring and an ideal string: Both are devoid of mass.
>
> Distinguishing characteristic of an ideal spring and an ideal string: An ideal string is inextensible, whereas the helical nature of a spring allows for changes in its length.

Spring force

This is the force of restoration generated within a spring.

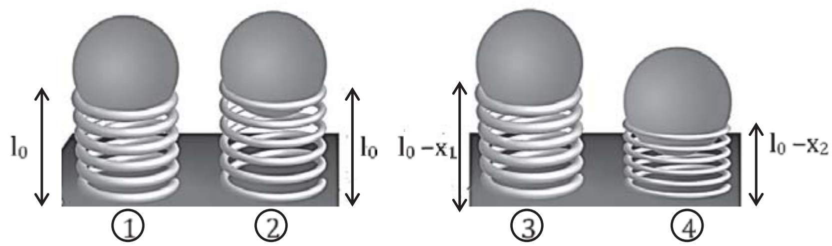
It is given by Hooke's law $\vec{F} = -k\vec{x} = -kx\hat{x}$, where $\vec{x}$ is the alteration in the spring's length, and k represents the spring constant.

The spring force always acts in the opposite direction to the applied force or the displacement direction. Therefore, it is considered negative.

In the case of an ideal spring (massless), the restoring spring force is, $F_{spring} = F_{ext}$

Spring constant (k):

This parameter represents the stiffness of the spring, indicating the level of difficulty in deforming it. It serves as a measure of the spring's inertia to withstand compression or extension. In the illustration, identical balls are placed on two springs of equal length, yet the change in length differs.



Here, spring force on spring 1 = spring force on spring 2

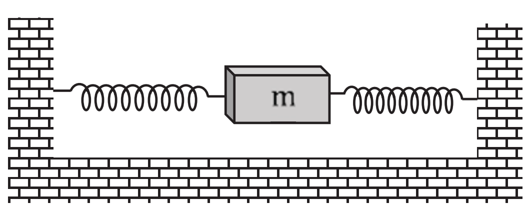
$$F_1 = F_2$$
$$k_1(x_1) = k_2(x_2)$$
$$k_1 > k_2 (\because x_1 < x_2)$$

Therefore, when an identical force is applied to two springs of the same length, the spring with a higher spring constant will experience less elongation or compression compared to the spring with a lower spring constant. A higher spring constant (k) indicates greater stiffness and robustness in the spring, making it more resistant to compression or extension.

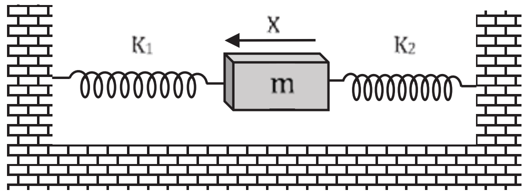
Spring balance:

It gauges the restoring spring force, equivalent to the tensile force acting on the spring. This tensile force is equal to the weight of the suspended body only when the system is in equilibrium. In the depicted illustration, when a mass is suspended from the vertical spring and the system attains equilibrium, the tension in the string equals the weight of the hung object.

Ex. Both springs illustrated in the diagram are in their natural (unscratched) state. If the block is displaced by a distance x and then released, what will be the initial acceleration?

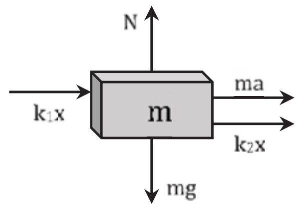


Sol. Illustrate the Free Body Diagram (FBD) of the mass m, as depicted in the figure. In this scenario, the left spring undergoes compression, exerting a restoring force in the rightward direction to regain its original shape. Simultaneously, the right spring undergoes elongation and, in order to restore its position, applies force in the rightward direction as well.



Equating forces in horizontal direction,

$$k_1x + k_2x = ma$$
$$(k_1 + k_2)x = ma$$
$$a = \frac{(k_1 + k_2)x}{m}$$

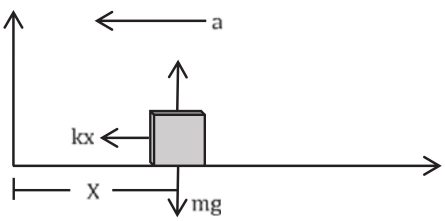


Ex. A particle with a mass of 0.3 kg experiences a force F = -kx, where k = 15 Nm^{-1}. What will be its initial acceleration when released from a position x = 20 cm?

Sol. Even though the nature of the particle is not explicitly stated as a spring, it is characterized by a spring constant, and the force is articulated in terms of spring force. Therefore, it is necessary to apply the concept of a spring to analyze the behavior of this particle.

From the FBD of the particle,

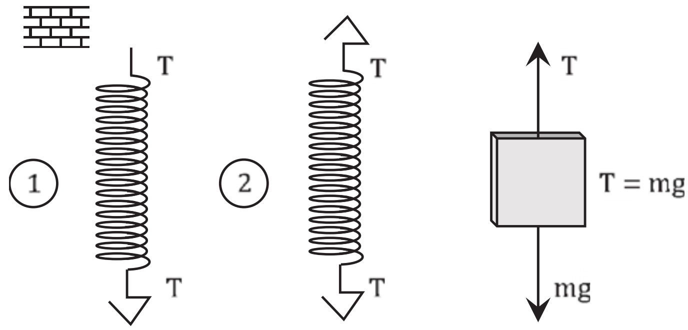
$$ma = kx$$
$$a = \frac{kx}{m} = \frac{15 \times 0.20}{0.3} = 10ms^{-2}$$



Ex. A 20 kg block is suspended using two lightweight spring balances, as depicted in the figure. Determine the following:

(a) Reading on spring balance 1 (b) Reading on spring balance 2

Sol. Create a Free Body Diagram (FBD) for the mass 'm' and the spring balances.

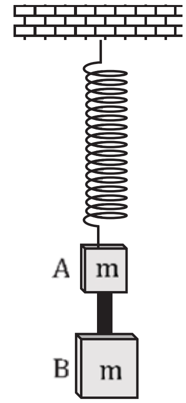


It is understood that the spring balance gauges the external force exerted upon it, represented as tension 'T' in this context. Consequently, both spring balances indicate their respective readings. $F_{spring\ 1} = F_{spring\ 2} = T = mg = 200$ Nor 20 kg on scales of the spring balances.

Ex. Two blocks, denoted as A and B, with equal mass 'm' and connected by a light string, are suspended by a spring as illustrated in the figure. Calculate the acceleration of both blocks immediately following the separation of the string.

(A) g, $\frac{g}{2}$ (B) g, g

(C) 2g, g (D) $\frac{g}{2}, \frac{g}{2}$



Sol. In this scenario, upon severing the string, block B will descend freely.
Therefore, either option (B) or (C) is accurate.
Next, let's determine the acceleration of block A.
Note: Restoring force in a spring does not change suddenly.

Before cutting of string, From FBD of B,

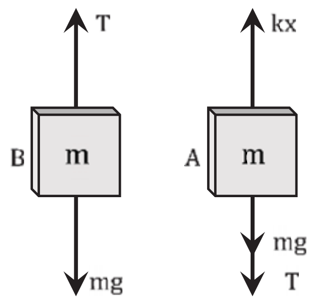
Tension in the string is equal to the weight of block B.

$$T = mg$$

From FBD of A,

$$F_{spring} = T + mg$$

$$F_{spring} = mg + mg = 2mg$$



Just after cutting of string,

The restoring force will stay constant, as it does not undergo sudden changes.

From FBD of A, $ma_1 = F_{spring} - mg$

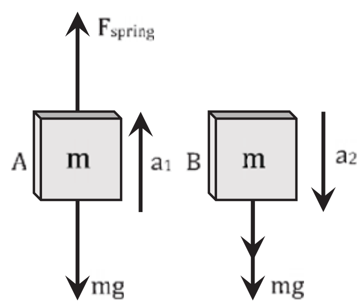
$$ma_1 = 2mg - mg$$

$$a_1 = g$$

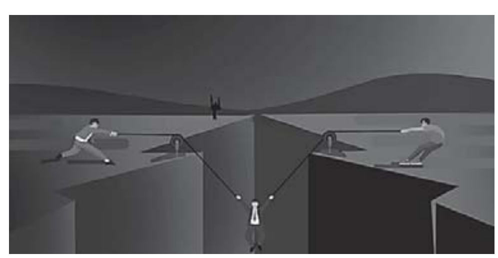
From FBD of B, $ma_2 = mg$

$$a_2 = g$$

Hence, option (B) is correct

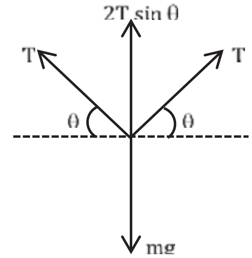


Ex. A man has fallen into a ditch with a width 'd,' and two of his friends are gradually pulling him out using a light rope and two fixed pulleys. Demonstrate that the force exerted by each friend on the road (assumed to be equal) increases as the man ascends. Determine the force when the man is at a depth 'h.'



Sol. The process occurs at a very slow pace, considered quasi-static. Consequently, we can assume equilibrium at each moment. Therefore, we can equate the forces for any component without taking into account the acceleration. ($a \rightarrow 0$).

Resolution of forces on hanging man, is shown in adjacent figure:



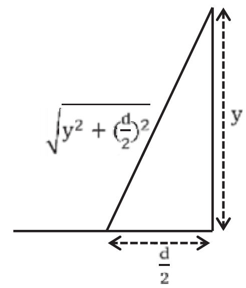
Balancing the forces in vertical direction,

$$2T\sin\theta = mg$$

$$T = \frac{mg}{2\sin\theta}$$

$$\sin\theta = \frac{y}{\sqrt{y^2 + (\frac{d}{2})^2}}$$

The value of sin θ at any given moment can be determined through geometric considerations. At any specific instant, assume the man is at a depth 'y.'



As the string has no mass, the tension in the string,

T = Force applied by each friend

T = F

Hence, the force exerted by each friend at the depth y = h is:

$$F = \frac{mg}{2\sin\theta} = \frac{mg}{\sqrt{h^2 + (\frac{d}{2})^2}} = \frac{mg\sqrt{h^2 + (\frac{d}{2})^2}}{2h}$$

As the man is lifted upward, the angle θ diminishes. Consequently, the value of sin θ also decreases, indicating that the force exerted by each friend (F) increases, as F is inversely proportional to sin θ.