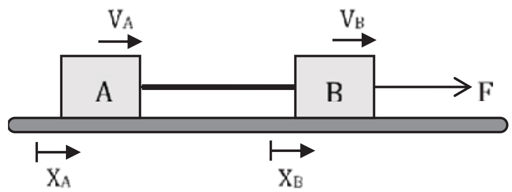
PSEUDO FORCE

Newton's Laws on a System

As the string cannot extend, we are bound by specific equations governing displacement, velocity, and acceleration, which are as follows:

$$x_A = x_B$$
$$\frac{dx_A}{dt} = \frac{dx_B}{dt} \Rightarrow v_A = v_B$$
$$\frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$$



Taking into account all the bodies as a unified system:

$$(\Sigma\vec{F}_{ext})_{sys,x} = (m_{sys}\vec{a}_{sys})_x$$
$$F = (m_A + m_B)a$$
$$a = \frac{F}{m_A + m_B}$$

What led us to select A and B collectively as a system?

Due to their identical acceleration, it is advantageous to treat them as a unified system.

Selection of system

Is it possible to regard objects with varying accelerations as a unified system?

Let's contemplate m_1, m_2, m_3, and so on as the masses of the system's objects, and $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the respective accelerations of the objects.

We can equilibrate the forces acting on each individual body by:

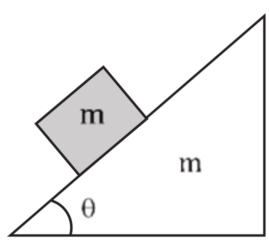
$$(\Sigma\vec{F}_{ext})_1 = m_1\vec{a}_1 \quad \text{... (1)}$$
$$(\Sigma\vec{F}_{ext})_2 = m_2\vec{a}_2 \quad \text{... (2)}$$
$$(\Sigma\vec{F}_{ext})_3 = m_3\vec{a}_3 \quad \text{... (3)}$$

Combining all the equations,

$$(\Sigma\vec{F}_{ext})_{sys} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 +$$

Therefore, we can treat the assembly of masses with distinct accelerations as a single system.

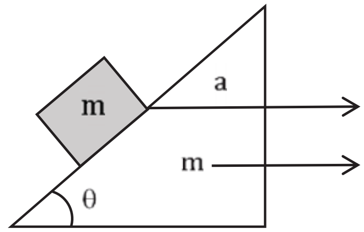
Ex. The mass m block slides on a wedge of mass m, which is free to move on the horizontal ground. Establish the relationship between the accelerations of the wedge and the block. (Assume all surfaces are frictionless)



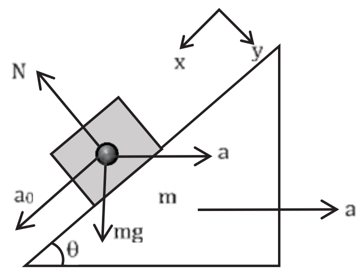
Sol. Comprehend the physical aspect: As the block descends along the surface of the wedge, the wedge moves in the direction to the right.

While the wedge progresses in the rightward direction, the block will follow suit. Consequently, the resultant velocity of the block will be the vector sum of these two velocities.

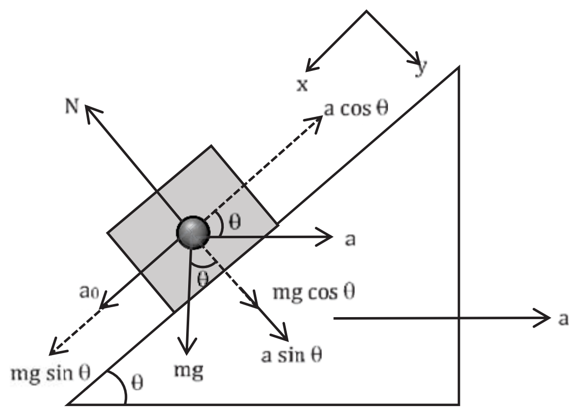
Consider 'a' as the acceleration of the wedge relative to the ground, and 'a_0' as the acceleration of the block relative to the wedge. In the frame of reference fixed to the ground, the acceleration of the block is the sum of these two accelerations.



Create a Free Body Diagram (FBD) for the block, indicating the forces acting on it, including weight and the normal reaction.



Resolve the components of weight and acceleration along the direction of the inclined surface and perpendicular to it.



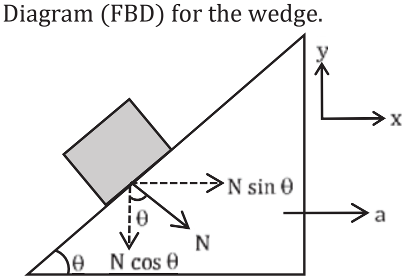
Equating forces in the 'x' direction $\qquad \Sigma \vec{F}_x = m\vec{a}_x$

$$mg\sin\theta = m(a_o - a\cos\theta) \qquad \dots (1)$$

Equating forces in the 'y' direction $\qquad \sum \vec{F}_y = m\vec{a}_y$

$$mg\cos\theta - N = m(a\sin\theta) \qquad \dots (2)$$

Now, create a Free Body Diagram (FBD) for the wedge.



Equating forces in the 'x' direction $\qquad \sum \vec{F} = m\vec{a}_x$

$$N\sin\theta = m(a). \qquad \dots (3)$$

Multiply the second equation by sinθ, then add the resulting equation to both equations (2) and (3). The result is.

$$mg\cos\theta\sin\theta = ma(1+\sin^2\theta)$$
$$g\cos\theta\sin\theta = a(1+\sin^2\theta) \qquad \dots (4)$$

Dividing (i) by (iv),

$$\frac{mg\sin\theta}{g\cos\theta\sin\theta} = \frac{m(a_o - a\cos\theta)}{a(1+\sin^2\theta)}$$

Eliminate similar terms, and through the process of solving.

$$a(1+\sin^2\theta) = a_o\cos\theta - a\cos^2\theta$$

Rearranging,

$$a + a\sin^2\theta + a\cos^2\theta = a_0\cos\theta$$

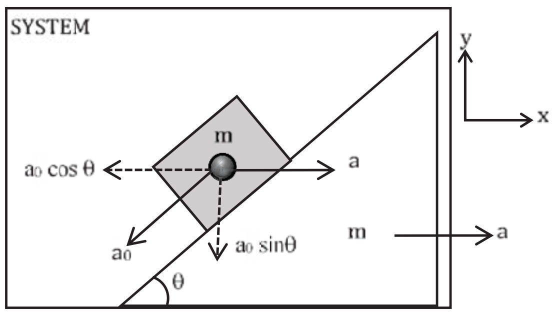
But we know that, $\sin^2\theta + \cos^2\theta = 1$

Factoring out 'a' and applying the above identity,

$$2a = a_o\cos\theta$$

Alternate Method

Considering the block and wedge together as a unified system,



Apply force equilibrium to this system;

Since there is no external force acting on the system in the x-direction,

$$\sum\vec{F}_x = m\vec{a}_x$$
$$0 = ma + m(a - a_o\cos\theta$$
$$2a = a_o\cos\theta$$

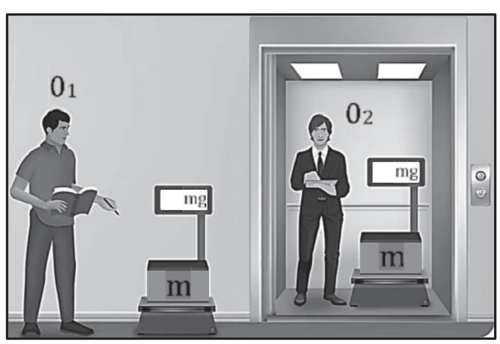
Therefore, a judicious choice of the system can considerably simplify our efforts.

Pseudo Force

Contemplate a block with mass 'm' positioned on a weighing machine within an elevator. Assume that observer O_1 observes this system from the perspective of the ground frame, while another observer, O_2, is situated inside the lift.

Case: 1 Elevator is at rest

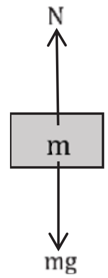
When the elevator is stationary, the reading on the weighing machine remains consistent for both observers.



Both observers depict the system using Free Body Diagrams (FBD).

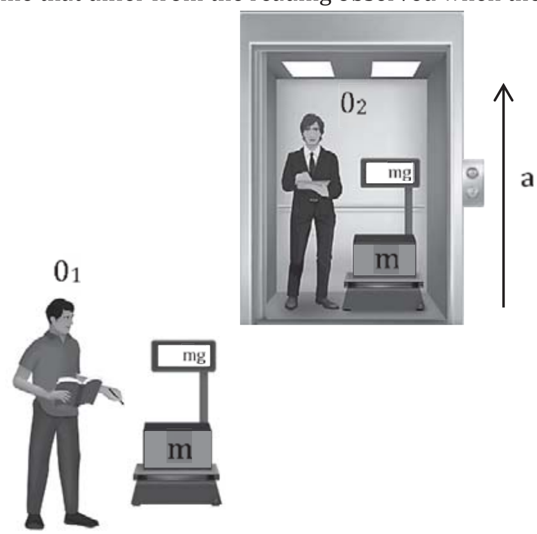
The forces exerted on the block include its weight (mg) and the normal reaction (N).

Along vertical direction, N = mg



Case: 2 Elevator is moving upwards with acceleration a

As the elevator begins ascending with an acceleration 'a,' both observers perceive readings on the weighing machine that differ from the reading observed when the elevator was stationary.



To comprehend the reason behind the discrepancy in readings under these circumstances, they employed physics concepts as outlined below:

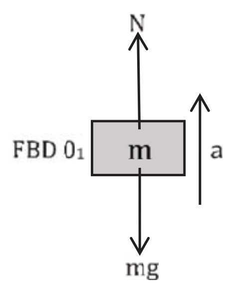
FBD of block for observer 0_1 standing in ground frame:

This observer notes that the system is ascending with an acceleration 'a.' Other forces acting on the block include its weight and the normal reaction.

In the vertical direction,

$$N - mg = ma$$
$$N = mg + ma$$
$$N = m(g + a)$$

Since the weighing machine gauges the normal reaction exerted upon it, the device will display a reading corresponding to $N = m(g + a)$.



FBD of block for observer O2 standing in lift frame:

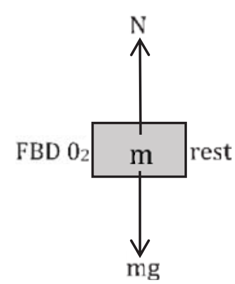
When the observer is inside the elevator, the block is stationary relative to them. Therefore, in this scenario,

The forces applied to the block include its weight (mg) and the normal reaction (N).

In the vertical dimension,

$$N = mg$$

However, according to observer O_2, the measurement displayed on the weighing machine corresponds to m (g + a).



Unable to justify his observation due to a distinct derivation, he sought to identify the discrepancy in this situation by employing the concept of relative motion.

Let $\vec{a}_{B,\,elevator}$ as the acceleration of the block in relation to the elevator. $\vec{a}_{B,G}$ block in relation to the ground and be the acceleration of the block relative to the ground, and $\vec{a}_{elevator,\,G}$denote the acceleration of the elevator relative to the ground.

$$\vec{a}_{B,\,elevator} = \vec{a}_{B,G} - \vec{a}_{elevator\,,G}$$

Multiplying each side by m.

$$m\vec{a}_{B,\,elentar} = m\vec{a}_{B,G} - m\vec{a}_{elivutor\,,G}$$

The normal reaction $(m\vec{a}_{B,\,elenatar})$ is the sum of $(m\vec{a}_{B,G})$ and $(-m\vec{a}\vec{a}_{elersator\,,G})$ when the elevator is in motion with acceleration.

Here $(m\vec{a}_{B,G})$ is the weight of the block and the term $(-m\vec{a}_{elevator\,,G})$ appears due to the accelerated frame. This can be articulated in force-related terms as:

$$(\sum\vec{F}_p)_{elevator} = (\sum\vec{F}_p)_{Ground} + (-m\vec{a}_{elevator\,,G}).$$

This additional force term $(-m\vec{a}_{elevator\,,G})$ appears from the mass of the block and the elevator's acceleration, which lacks physical significance.

Non-Inertial Frames

The variation in the net force within the ground and elevator frames is ascribed to the acceleration of the elevator. If the elevator were moving with a constant velocity, this term would be negligible. Thus

It pertains not to the mere motion of the elevator but rather to its acceleration. Frames undergoing acceleration are termed non-inertial frames of reference.

In frames of reference experiencing acceleration, Newton's law appears to be inadequate.

Hence, Newton's law is not applicable in non-inertial frames of reference.

The additional term is artificial and referred to as a pseudo force.

Thus,

$$\vec{F}_{pseudo} = -m\vec{a}_{elevator\,,G}$$

Therefore, for non-inertial frames, Newton's law can be adjusted as follows:

$$(\sum\vec{F}_{ext})_{elevator} = (\sum\vec{F}_{ext})_{Ground} + \vec{F}_{pseudo}$$

The inertial forces, commonly referred to as pseudo forces, are necessitated due to the utilization of non-inertial frames.

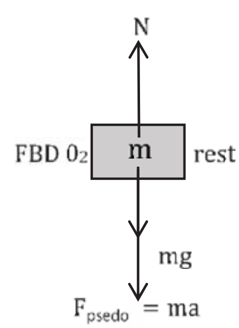
$$\vec{F}_{pseudo} = -m_{sys}\,\vec{a}_{non\text{-}inertial\ frame}$$

Properties of pseudo force:

It's a fictitious force, meaning it lacks a corresponding reaction pair.

Its direction is consistently opposite to the acceleration of the frame of reference.

Therefore, the Free Body Diagram (FBD) for a mass moving with acceleration in the elevator frame is adjusted to:

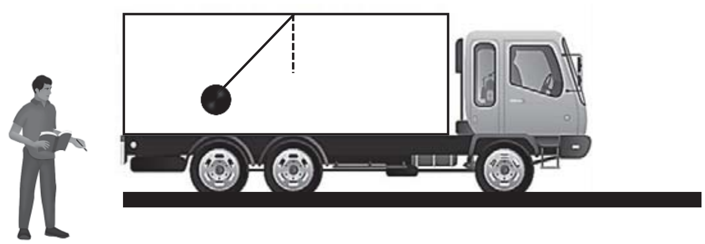


Equating the forces in the vertical direction,

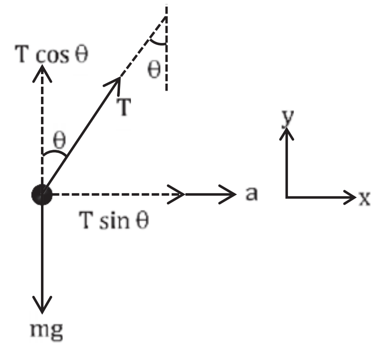
$N = mg + ma$

$N = m(g + a)$

Ex. The illustration depicts a pendulum hanging from the roof of a truck that experiences a constant acceleration 'a' relative to the ground. Determine the deviation of the pendulum from the vertical, observed in both the ground frame and the frame attached to the truck.



FBD in ground frame:



Balancing forces $\Sigma \vec{F}_x = m\vec{a}_x$

$T\sin\theta = ma$ ... (1)

In y-direction, $\Sigma \vec{F}_y = m\vec{a}_y$

$T\cos\theta = mg$ ... (2)

Dividing equation(i) by (ii),

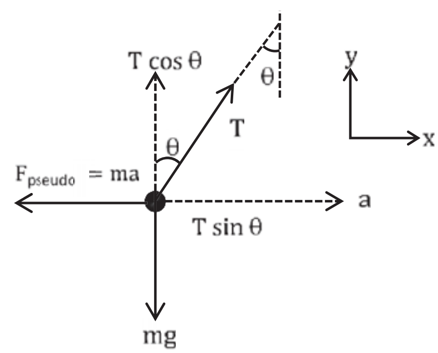
$$\tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Now, if we address the problem within the frame of the truck, consider the observer situated within the truck.

Balancing forces in horizontal direction,$\vec{F}_{pseudo} = -m_{object}\,\vec{a}_{frame}$

Balancing forces in horizontal direction



$\Sigma \vec{F}_x = m\vec{a}_x$

$T\sin\theta = ma$ ... (1)

In y-direction, $\Sigma \vec{F}_y = m\vec{a}_y$

$T\cos\theta = mg$ ... (2)

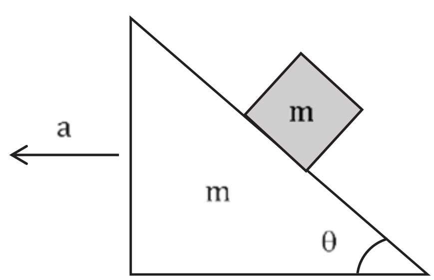
Dividing equation(i) by (ii),

$$\tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}\left(\frac{a}{a}\right)$$

Therefore, the deflection remains consistent in both frames of reference.

Ex. Consider a block with mass 'm' resting on a wedge inclined at an angle θ, as illustrated in the figure. The wedge is subjected to an acceleration 'a'. Determine the minimum value of 'a' required for the mass 'm' to fall freely.



Sol. For the block to fall freely, the normal reaction between the wedge and the block should be zero (N = 0). Let's analyze this scenario in a non-inertial (wedge) frame. The components depicted in blue correspond to the weight force, while the components in yellow represent the pseudo force.

$$\Sigma\vec{F}_y = m\vec{a}_y$$

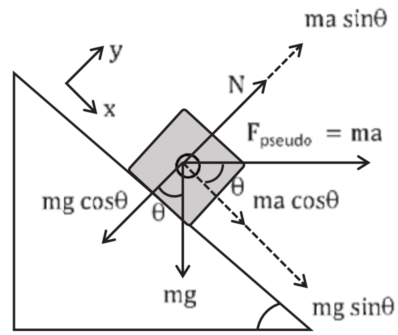
$$N + ma\sin\theta = mg\cos\theta$$

As N = 0,

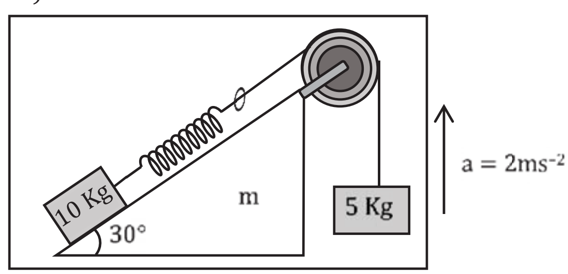
$$ma\sin\theta = mg\cos\theta$$

$$a = g\frac{\cos\theta}{\sin\theta}$$

$$a = g\cot\theta$$



Ex. In the depicted diagram, the measurement on the spring balance (N) can be determined as follows: (Assume g = 10 ms^{-2}).



Sol. The spring balance gauges the force applied to the spring. In this case, as the spring is considered massless, the measurement corresponds to the tension in the string.
Let's solve the problem in lift frame,
Let 'a_0' denote the acceleration of the blocks relative to the wedge. As the 5 kg block descends, the 10 kg block moves along an inclined plane with an acceleration identical to that of the 5 kg block, owing to the inextensibility of the string.
The tension in the spring equals the tension in the string, given that they are both ideal.

$$T_{spring} = kx = T$$

From the FBD of wedge and mass

Balancing force in x-direction

$$T - m_1 g\sin 30° - m_1 a\sin 30° = m_1 a_0$$

$$T - (10)(10)\frac{1}{2} - (10)(2)(\frac{1}{2}) = 10a_0$$

$$T - 60 = 10a_o \qquad \dots (1)$$

For 5 Kg block, along vertical direction,

$$m_2 g + F_{pseudo} - T = m_2 a_o$$

Where, $F_{pseudo} = m_2 a = 10N$

$$(50 + 10) - T = 5a_o$$

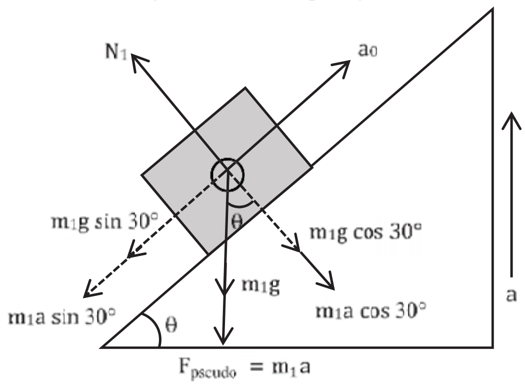
$$60 - T = 5a_o \qquad \dots (2)$$

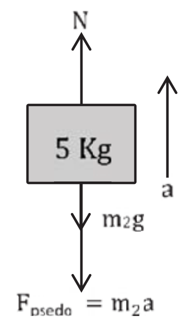
Adding equation(1) and (2),

$$0 = 15a_o$$
$$a_o = 0$$

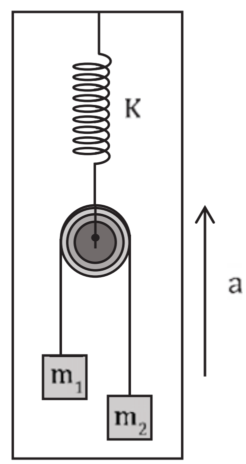
Thus,

Tension in string $=$ Force in spring $= T = 60$ N





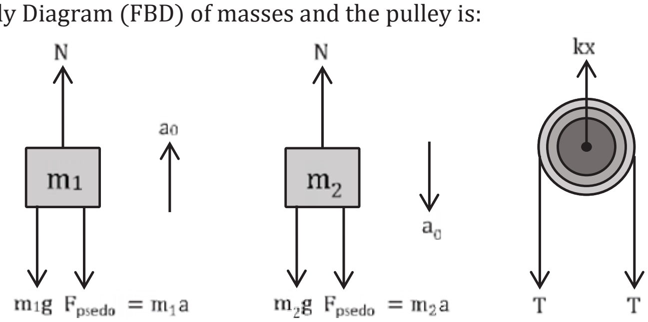
Ex. A pulley system with two blocks is suspended from the ceiling of an elevator ascending with an acceleration 'a.' Determine the deformation in the spring.



Sol. Solving in lift frame,

Let 'a_0' denote the acceleration of the blocks with respect to the elevator.

The Free Body Diagram (FBD) of masses and the pulley is:



Given the lightweight nature of the string and spring, we have the relationship kx = 2T.

For first block,

Equating forces in the vertical direction,

$$T - m_1g - m_1a = m_1a_o \qquad \text{... (1)}$$

For second block,
Balancing forces in vertical direction,

$$m_2g + m_2a - T = m_2a_o \quad \text{... (2)}$$

Adding two equations

$$(m_2 - m_1)g + (m_2 - m_1)a = (m_2 + m_1)a_o$$

$$\frac{(m_2 - m_1)(g + a)}{m_1 + m_2} = a_o$$

Substituting value of a_0 in equation (1),

$$T = m_1[\frac{(m_2 - m_1)(g + a)}{m_1 + m_2}] + m_1(g + a)$$

$$T = \frac{m_1m_2(g + a) - m_1^2(g + a) + m_1^2(g + a) + m_1m_2(g + a)}{m_1 + m_2}$$

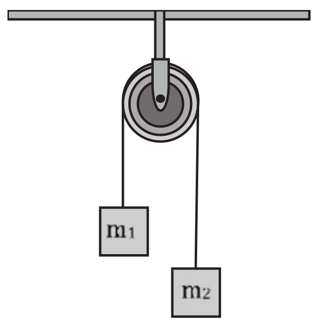
$$T = \frac{2m_1m_2(g + a)}{m_1 + m_2}$$

Alternate Method

Standard Atwood machine,
We have obtained the formula for the acceleration and tension in this apparatus,

$$a = \frac{(m_2 - m_1)g}{(m_2 + m_1)}$$

$$T = (\tfrac{2m_1m_2}{m_1+m_2})g \quad \text{... (3)}$$



The method of using the pseudo-force concept can be replaced by employing the effective acceleration due to gravity. The effective acceleration due to gravity varies in different cases, and its values are as follows.

When body is moving up with acceleration a	$g_{eff} = g + a$
When body is moving down with acceleration a	$g_{eff} = g - a$

Since the elevator is ascending, the acceleration would undergo a modification to

$$g \to g_{eff} = g + a$$

Put in (3), we get,

$$T = (\frac{2m_1m_2}{m_1 + m_2})(g + a)$$

As we know

$$2T = kx$$

$$x = \frac{2T}{k}$$

$$x = \frac{4m_1m_2(g + a)}{(m_1 + m_2)k}$$

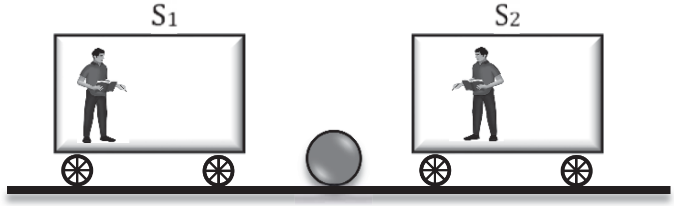
Ex. The magnitudes of the particle's accelerations observed from two frames, S_1 and S_2, are equal, each measuring 4 ms^{-2}. Consequently,

(A) The frames must be stationary relative to each other.

(B) The frames may be in motion relative to each other, but neither should experience acceleration with respect to the other.

(C) The acceleration of S_2 relative to S_1 may either be zero or 8 ms^{-2}.

(D) The acceleration of S_2 relative to S_1 may be any value between zero and 8 ms^{-2}.



Sol. We have,

$$\vec{a}_{p,s_1} = \vec{a}_p - \vec{a}_{s_1}$$
$$\vec{a}_{p,s_2} = \vec{a}_p - \vec{a}_{s_2}$$

Subtracting equations,

$$\vec{a}_{p,s_1} - \vec{a}_{p,s_2} = -\vec{a}_{s_1} + \vec{a}_{s_2}$$

Rearranging above equation

$$\vec{a}_{s_2,s_1} = \vec{a}_{s_2} - \vec{a}_{s_1} = \vec{a}_{p,s_1} - \vec{a}_{p,s_2}$$

Considering this involves vector addition, the resulting magnitude after the addition will be minimal, corresponding to the difference between the magnitudes of the vectors, and maximal, reflecting the sum of their magnitudes.

$$0 \le |\vec{a}_{s_2,s_1}| \le 8$$

Hence option (D) is correct.

Concept of Pseudo Force

What are non-inertial frame of references?

Frames of reference that undergo acceleration concerning the ground or any inertial frame are termed non-inertial frames.

What is a pseudo force?

A pseudo force, also referred to as a fictitious force, inertial force, or d'Alembert force, is a seeming force that influences all masses whose motion is explained within a non-inertial frame of reference. The pseudo force's magnitude is determined by multiplying the mass of an object by the acceleration of the frame of reference.

Ex. Upon passing through a plank with a thickness h, a bullet transitioned its velocity from v_o to v. Calculate the duration of the bullet's motion within the plank, with the assumption that the resistance force is proportionate to the square of the velocity.

Sol. Resistance force, $F_R \propto V^2$

$$F_R = -kv^2$$

Here, k is proportionality constant

The negative sign in the preceding equation signifies that the resistance force is exerted in the direction opposite to the bullet's velocity.

We know that

$$F_R = ma$$
$$ma = -kv^2$$

Acceleration of bullet $$a = \frac{-kv^2}{m}$$

$$\frac{dv}{dt} = \frac{-kv^2}{m}$$

Integrating from time, t = 0 to t = t

$$\int_{V_o}^{v} \frac{dv}{V^2} = -\frac{k}{m}\int_0^t dt$$

Integrating and putting limits,

$$\frac{1}{V} - \frac{1}{V_o} = \frac{kt}{m}$$

$$t = \frac{m(V_o - V)}{kV_oV} \quad \text{... (1)}$$

Also, acceleration can be written as follows

$$a = v\frac{dv}{dx}$$

The provided equation articulates acceleration by considering the alteration in velocity concerning displacement.

$$a = \frac{-kv^2}{m} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} \frac{dv}{V} = -\frac{k}{m}\int_0^h dx$$

$$\ln(v) - \ln(v_o) = -\frac{kh}{m}$$

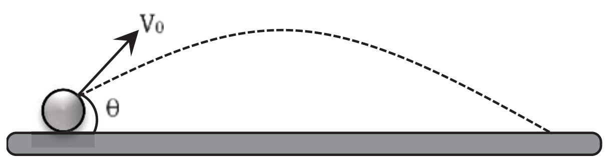
$$\ln(\frac{V}{V_o}) = -\frac{kh}{m}$$

$$k = \frac{m}{h}\ln(\frac{V_0}{V}) \quad \text{... (2)}$$

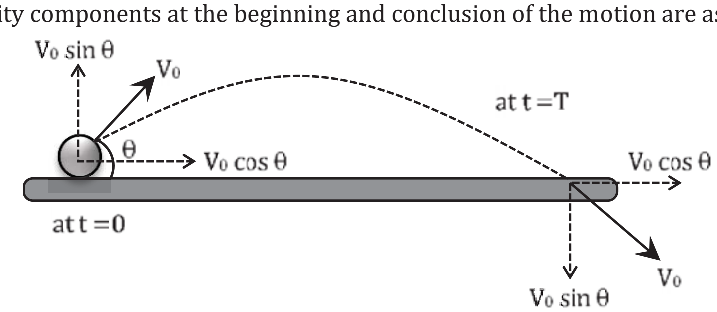
Put the value of k from equation (ii) into equation (i), We get,

$$t = \frac{h(V_o - V)}{VV_o\ln(\frac{V_o}{V})}$$

Ex. A mass m is projected at an angle θ to the horizontal with an initial velocity v0. Assuming negligible air drag, determine the magnitude of the momentum increment Δp over the entire duration of motion.



Sol. The velocity components at the beginning and conclusion of the motion are as follows:



Initial momentum, $\vec{p}_i = (mV_o\cos\theta)\hat{i} + (mv_o\sin\theta)\hat{j}$

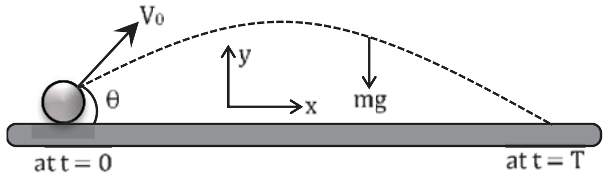
Final momentum, $\vec{p}_f = (mv_o\cos\theta)\hat{i} - (mv_o\sin\theta)\hat{j}$

Subtracting the final momentum from initial,

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -(2mV_o\sin\theta)\hat{j}$$

Hence the magnitude $|\Delta\vec{p}| = 2mv_o\sin\theta$

Alternative way



From Newton's second law of motion,

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{ext} = -(mg)\hat{j}$$

Note: When F_{ext} is constant, the average rate of change in momentum is equivalent to the instantaneous rate of change in momentum.

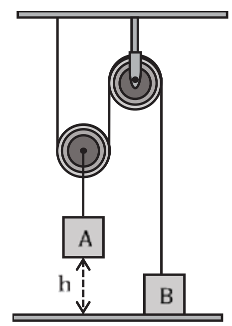
$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} = \frac{\Delta p}{\Delta t} = -mg\hat{j}$$

Considering the magnitude, $\Delta p = mg\Delta t$

$$\Delta p = mg(\frac{2v_o \sin\theta}{g})$$

$$\Delta p = 2mv_o \sin\theta$$

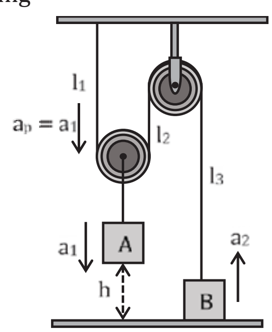
Here, time of flight is given by, $\Delta t = \frac{2V_o \sin\theta}{g}$



Ex. In the depicted setup, the mass of body A is n = 4 times greater than that of body B. The height is h. assuming ideal conditions, at a particular moment, body B is released from the ground, initiating the motion of the arrangement. What is the maximum height that body B will reach?

Sol. Note: The assumption of ideal conditions implies that all pulleys and strings are considered ideal. In the scenario, initially, block B is at ground level, but due to the greater weight of block A,

it descends while block B ascends. Variables for acceleration and string lengths are assumed. Block A and the pulley will descend with identical accelerations.



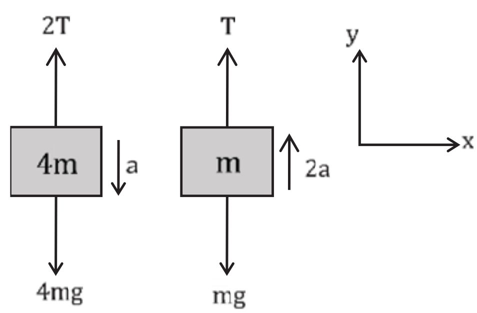
Equation for the constraint relation,

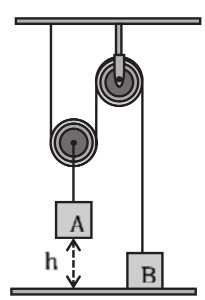
$$\ddot{I}_1 + \ddot{I}_2 + \ddot{I}_3 = 0$$

$$a_1 + a_1 - a_2 = 0$$

$$a_1 = a; \quad a_2 = 2a$$

Now, create Free Body Diagrams (FBDs) to establish force equilibrium.





For block A,

Force balance in vertical direction,

$$4mg - 2T = 4ma$$

$$2mg - T = 2ma \quad \text{... (1)}$$

For block B, $T - mg = 2ma$ ... (2)

Adding both the equation (1) and equation (2), we get,

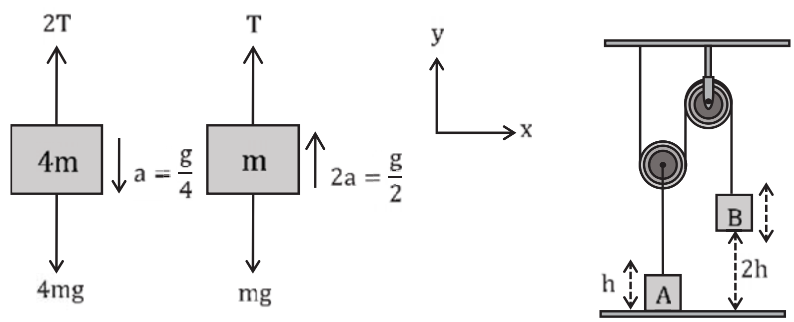
$$a = \frac{g}{4}$$

$$a_1 = \frac{g}{4}$$

$$a_2 = 2a_1 = \frac{g}{2}$$

Right before the collision of block A with the ground

> The constrained relationship between velocity and displacement is identical to that of the constrained relationship between acceleration. Let v represent the velocity of block A just before it reaches the ground.



$$V_A = V$$

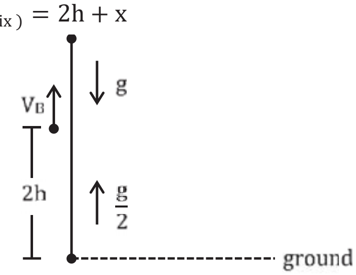
$$V_B = 2V$$

Upon the impact of block A with the ground, $x_A = h$ and $x_B = 2h$

Upon A's impact with the ground, the upward movement of B persists due to the inertia of block B, resulting in the slackening of the string.

For B

Maximum height of B, $H_{B(mix)} = 2h + x$



> Acceleration of block B up to the specified height (2h) is $\frac{g}{2}$. Beyond a height of (2h), the string becomes slack, indicating the absence of tension. Consequently, the acceleration of block B after reaching the height (2h) is -g.

Hence,
For height up to (2h),

$$V_B^2 = 0 + 2 \times \frac{g}{2} \times 2h$$
$$V_B^2 = 2gh$$

Using kinematics equation

$$v^2 = u^2 + 2as$$

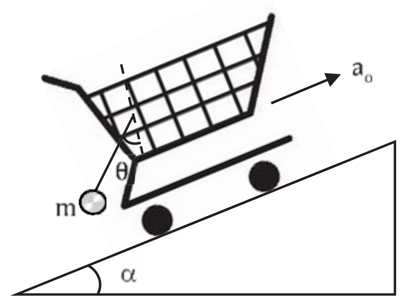
After height (2h),

$$0 = V_B^2 - 2 \times g \times x$$
$$x = \frac{V_B^2}{2g}$$
$$x = \frac{2gh}{2g}$$
$$x = h$$

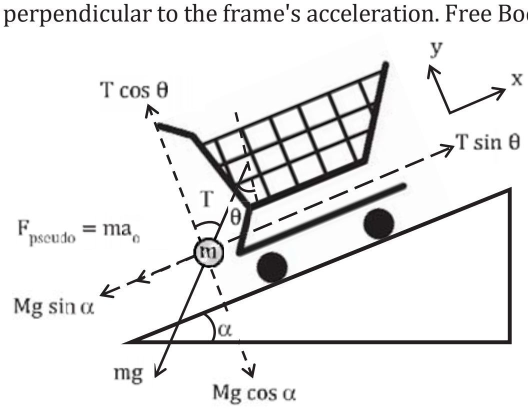
The maximum attainable height for the block is x + 2h.

$$H_{B_{max}} = 2h + h$$
$$H_{B_{max}} = 3h$$

Ex. A pendulum with mass m is suspended from a support attached to a trolley. Determine the direction of the string as the trolley ascends on an inclined plane with an acceleration of a_0 (Note: The string and bob remain fixed with respect to the trolley).



Sol. Let's transition into the trolley frame. Since it is a non-inertial frame, we must introduce a pseudo force in a direction perpendicular to the frame's acceleration. Free Body Diagram (FBD) of mass m,



Trolley frame (Non-inertial frame)

> The pendulum is at rest or in equilibrium within the trolley frame. Consequently, the net force (F_{net}) on the pendulum in the trolley frame is zero.

Force balance along the x-axis,

$$T\sin\theta = ma_o + mg\sin\alpha \quad \dots (1)$$

Force balance along the y-axis,

$$T\cos\theta = mg\cos\alpha \quad \dots (2)$$

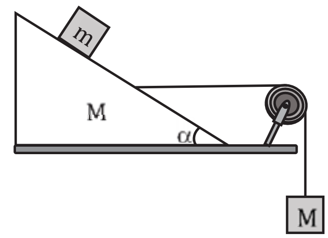
Divide equation (1) by (2),

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m(a_o + g\sin\alpha)}{mg\cos\alpha}$$

$$\tan\theta = \frac{a_o + g\sin\alpha}{g\cos\alpha}$$

$$\theta = \tan^{-1}\left(\frac{a_o + g\sin\alpha}{g\cos\alpha}\right)$$

Ex. Determine the mass (M) of the suspended block in the given figure that will prevent the smaller block from slipping over the triangular block. All surfaces are frictionless, and the strings and pulleys are weightless.

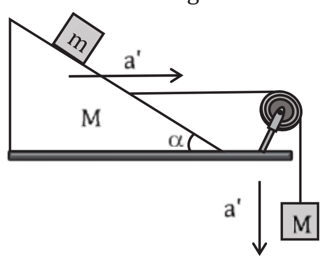


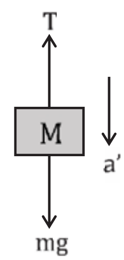
Sol. Let a' represent the acceleration of blocks M and M' relative to the ground.

Free Body Diagram (FBD) of the block with mass M,

Equating forces in the vertical direction,

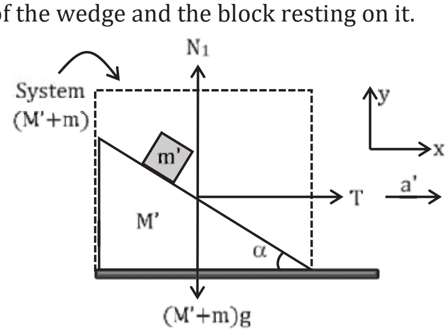
$$Mg - T = Ma' \quad \dots (1)$$





Free Body Diagram (FBD) of the system,

Choose the system in a way that the only external forces include the tension, normal reaction, and the combined weight of the wedge and the block resting on it.



Balancing forces in horizontal direction,

$$T = (M' + m)a' \quad \dots (2)$$

Add equation (1) and equation (2)

$$a' = \frac{Mg}{M' + m + M} \quad \dots (3)$$

Wedge frame (Non-inertial)

To prevent block m from slipping, the net force on the block along the incline should be zero. Since there is no slipping between the block and the wedge, the block with mass m remains at rest in the frame of the wedge.

Equating forces in the horizontal direction,

$$\sum F_x = 0$$
$$ma' \cos\alpha = mg\sin\alpha$$
$$a' = g\tan\alpha$$

Substituting the acceleration value from equation (3),

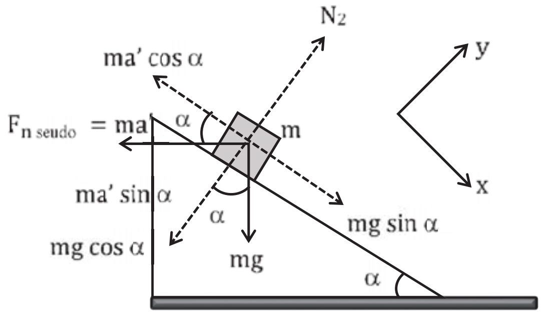
$$a' = \frac{Mg}{M + M' + m} = g\tan\alpha$$

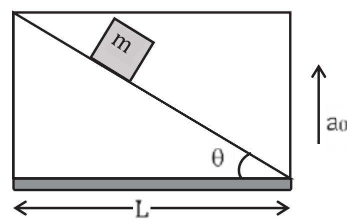
Rearranging,

$$\frac{M}{\tan\alpha} = M + M' + m$$
$$M\cot\alpha = M + M' + m$$
$$M(\cot\alpha - 1) = M' + m$$

Thus, Mass M is written as follows

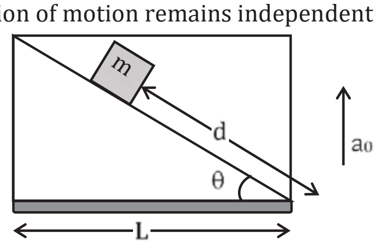
$$M = \frac{M' + m}{\cot\alpha - 1}$$





Ex. A particle descends a frictionless inclined plane inclined at an angle θ fixed in an elevator ascending with an acceleration a_0. The base of the incline has a length L. Determine the time taken by the particle to reach the bottom.

Sol. It is known that the duration of motion remains independent of the frame of reference.



In elevator frame (Non-inertial frame)

$$\cos\theta = \frac{L}{d}$$
$$d = \frac{L}{\cos\theta}$$

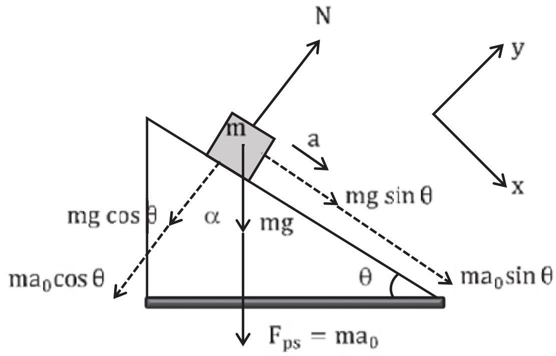
Consider a as the acceleration of block m relative to the ground.

Equating forces in the horizontal direction,

$$\sum \vec{F}_x = m\vec{a}_x$$
$$mg\sin\theta + ma_o\sin\theta = ma$$
$$a = (g + a_o)\sin\theta$$

Time remains independent of the frame of reference. The block starts with an initial velocity of zero. Since the acceleration is constant, kinematic equations hold true.

$$x = ut + \frac{1}{2}at^2$$



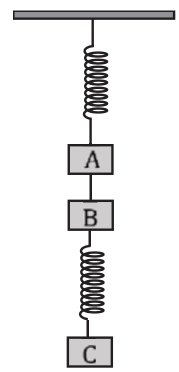
Total distance covered is as follows:

$$\frac{L}{\cos\theta} = \frac{1}{2} \times (g + a_o)\sin\theta \times t^2$$

(The block starts from rest, u = 0.)

$$t^2 = \frac{2L}{\sin\theta\cos\theta(g + a_o)}$$

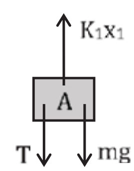
$$t = [\frac{2L}{\sin\theta\cos\theta(g + a_o)}]^{\frac{1}{2}}$$



Ex. The system depicted in the figure is in a state of equilibrium. Determine the acceleration of blocks A, B, and C at the moment when the spring between B and C is severed. (Assume the springs are ideal, and the mass of each block is m.)

Sol. The spring force undergoes no instantaneous alteration. However, there is an immediate change in tension within the string.

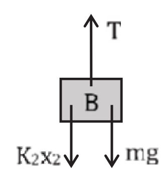
Pre-spring cut



FBD of block A,

Equating forces in the vertical direction,

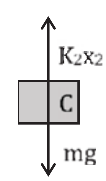
$$k_1x_1 = T + mg \quad \dots (1)$$



FBD of block B,

Equating forces in the vertical direction,

$$T = k_2x_2 + mg \quad \dots (2)$$



FBD of block C,

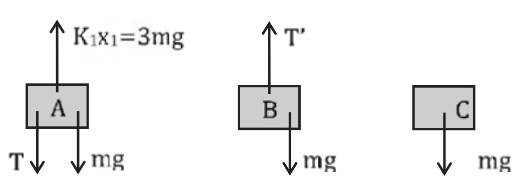
Equating forces in the vertical direction,

$$k_2x_2 = mg \quad \dots (3)$$

Adding equations (1), (2), and (3), we get,

$$k_1x_1 = 3mg$$

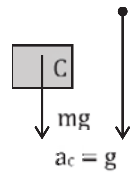
Post-spring cut



For block C

$$mg = ma_c$$
$$a_c = g$$

Acceleration of block C is g in downward direction



For block A and block B,

Applying force equilibrium to the combined bodies and utilizing $k_1x_1 = 3mg$,

$$3mg - (mg + mg) = (m + m)a$$

Rearranging,

$$a = \frac{g}{2}$$