SECOND LAW QUANTITATIVE ANALYSIS

First law:

An object remains in a state of inertia unless an external force is applied to it.

Second law:

The magnitude of a net unbalanced force is directly proportional to the rate at which the body's momentum changes.

Third law:

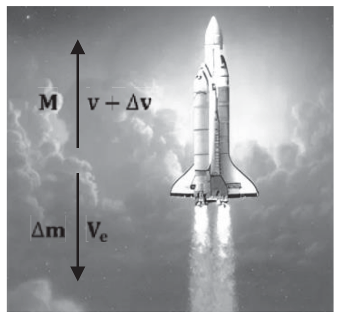
Forces in nature always occur in pairs.

Quantitative Understanding of Newton's Third Law

Imagine a rocket with a mass of M being launched at a speed v. When its velocity increases to $v + \Delta v$, the gases from the exhaust fuel, with a mass of Δm, move downward with a velocity v_e.

In adherence to Newton's third law of motion,

$$(M + \Delta m)V = M(V + \Delta V) + \Delta m(V - V_e)$$

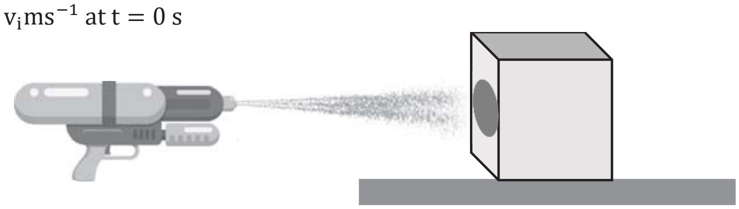


To comprehend Newton's second law, let us conduct a series of experiments.

Imagine a force gun pushing a block, causing it to move a certain distance. Let's examine what happens at different points during this motion.

First instance

Initial velocity $= v_i ms^{-1}$ at $t = 0$ s



Second instance

Final velocity $= v_f ms^{-1}$ at $t = Ts$

Let, Force applied = F N

Mass of the block = M kg

Average acceleration of the block will be,

$$a_{avg} = \frac{(V_f - V_i)}{T}$$

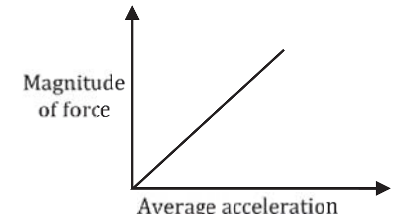
To understand how force, mass, and acceleration are connected, let's change each of these factors one at a time through experiments.

Experiment 1

Change the force while keeping the block's mass the same. Measure the average acceleration during the same time. It's noticed that the force is directly related to the measured average acceleration. This relationship can be illustrated in the adjacent figure.

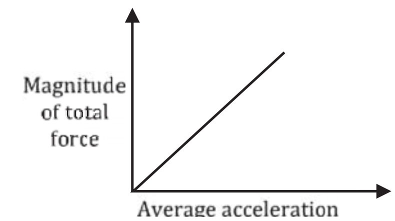
To understand how force, mass, and acceleration are connected, let's change each of these factors one at a time through experiments.

$$F_{mag} \propto \text{Average acceleration (a)}$$



Experiment 2

Use multiple force guns from one direction. Ensure they are all in the same direction to add the forces vector ally. Keep the block's mass constant and measure the average acceleration for the same time period. In this scenario, the total force will be the vector sum of all the applied forces. It is noticed that this total force is also proportionate to the magnitude of the average acceleration.



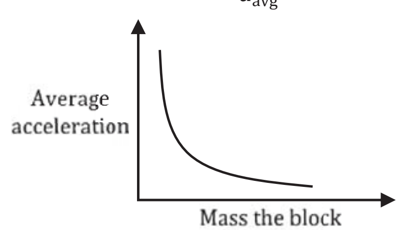
Vector sum of forces is directly proportional to the average acceleration, i.e.,

$$\Sigma F_{mag} \propto \text{Average acceleration (a)}$$

Experiment 3

Maintain a constant force magnitude and change the mass of the block. Measure the average acceleration for the same time period. In this case, we notice the following:
The average acceleration varies inversely with the mass of the block.

$$M_{block} \propto \frac{1}{a_{avg}}$$



Experiment 4

Change the direction from which the force gun is fired. Keep the block's mass constant and observe the direction of acceleration. It is noticed that the directions of both the net force and acceleration align.

Summary of experiments

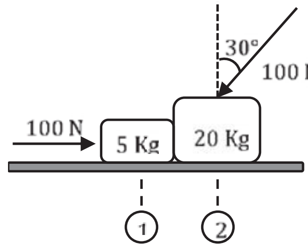
The condensed findings from the experiments above aid us in understanding the connection between force, mass, and acceleration.

$$F_{mag} \propto a_{avg}$$
$$\sum F \propto a_{avg}$$
$$M_{block} \propto \frac{1}{a_{avg}}$$

The net force and acceleration share the same direction.
Thus, the relation can be written in vector form as,

$$\sum \vec{F} \propto M\vec{a}$$

Ex. Two blocks, in contact with each other, encounter a force of 100 N each, as depicted. Determine the acceleration of the 5 kg block. (Assume smooth surfaces.)

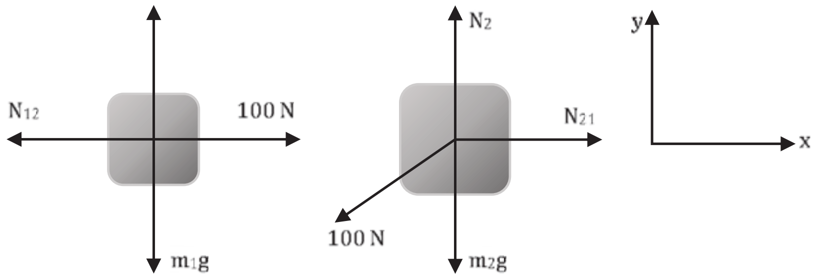


Sol. In this case, the surface is frictionless. Therefore, friction can be disregarded.

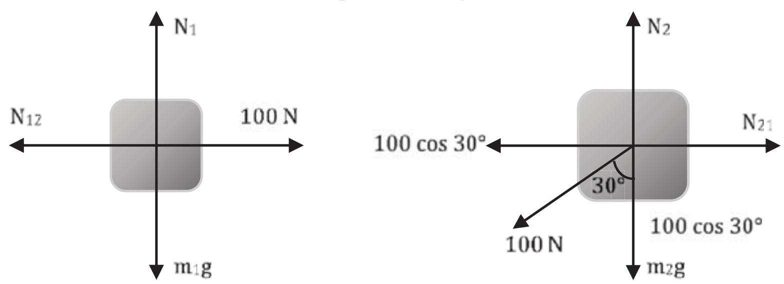
Step 1: Sketch the Free Body Diagram (FBD). Consider the 5 kg block as having a mass of m_1 and the 20 kg block as having a mass of m_2. The 5 kg block experiences a total of four applied forces.

1. Weight in downward direction
2. 100 N force in positive x direction
3. Normal reaction of body 2, N_{12}
4. There will be a normal reaction N_1 from the ground as well.

For the second body, 100 N force is applied at 30o inclined from vertical.



Step 2: Establish the axes and break down the forces into their components along these axes. Choose axes in a way that aligns a maximum number of forces along them, minimizing the need for force resolution. Resolve the 100 N inclined force along these two selected directions, as illustrated in the provided figure.



Equilibrium is maintained in the y-direction. Consequently, both bodies will undergo motion in the x-direction with an acceleration denoted as 'a'.

Step 3: Formulate the equations of motion along the axes and proceed with their resolution.

Take the positive x-axis direction along the acceleration For 5kg, block

Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$100 - N_{12} = (5)a \quad \text{... (1)}$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_1 - m_1 g = 0 \quad \text{... (2)}$$

For 20**kg** block

Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$N_{21} - 50 = (20)a \quad \text{... (3)}$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_2 - m_2 g - 100\cos 30^\circ = 0 \quad \text{... (4)}$$

As the two block apply the same normal reaction on each other.

$$|N_{12}| = |N_{21}|$$

On adding equations (1) and (3), we get

$$100 - 50 = 25a$$
$$50 = 25a$$
$$a = 2m^{-2}$$

Ex. Depicted is a setup with a two-block system. Determine the overall external force affecting the 1 kg and 2 kg blocks, respectively. (Assume the surfaces are frictionless.)

(A) 4 N, 8 N (B) 1 N, 2 N (C) 2 N, 4 N (D) 3 N, 6 N

Sol. Since both blocks will move together under the applied force, the acceleration of both blocks will be identical.

Step 1: Draw FBD,

FBD for the blocks are shown in the figure.

For block 1,

Along x-direction $\sum \vec{F}_x = m\vec{a}_x$

$$6 - N_{12} = (1)a \quad \text{... (1)}$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_1 = 1(10)$$
$$N_1 = 10N \quad \text{... (2)}$$

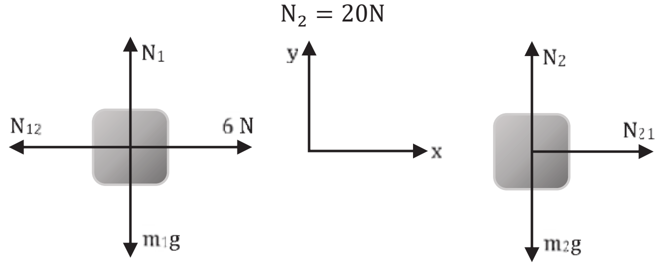
For body **2**

Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$N_{21} = (2)a \quad \text{... (3)}$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_2 = 20N \quad \text{... (4)}$$



Since the two blocks exert equal normal reactions on each other,

$$|N_{12}| = |N_{21}|$$

On adding equation (1) and (3), we get.

$$6 = 3a$$
$$a = 2m^2$$

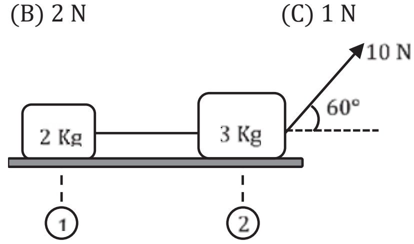
Force on block with mass 1kg = ma = 1 × 2 = 2 N

Force on block with mass 2 kg = ma = 2 × 2 = 4 N

Hence, option (C) is correct.

Ex. The illustration depicts two blocks linked by a light, inextensible string. A pulling force of 10 N is exerted on the larger block at an angle of 60° with the horizontal. Determine the tension in the string connecting the two masses.

(A) 5 N (B) 2 N (C) 1 N (D) 3 N



Sol. The string is both massless and inextensible. Consequently, the tension in the string will be uniform throughout its length. The Free Body Diagrams (FBD) for the two bodies are illustrated in the figure:

For body 1

Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$T = (2)a \quad \dots (1)$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_1 = 2(10)$$

$$N_1 = 20N \quad \dots (2)$$

For body 2,

Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$10\cos 60^\circ - T = (3)a$$

$$5 - T = (3)a \quad \dots (3)$$

Along y-direction, $\sum \vec{F}_y = m\vec{a}_y = 0$

$$N_2 + 10\sin 60^\circ = 3(10)$$

$$N_2 + 5\sqrt{3} = 30N \quad \dots (4)$$

On Adding equation (1) and (3), we get,

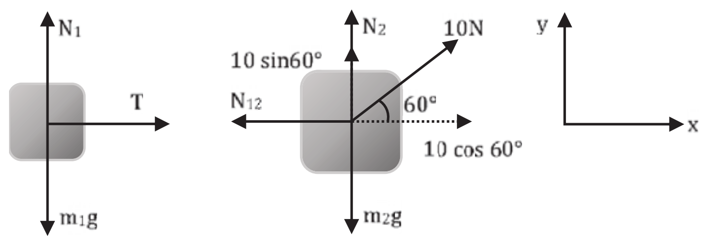
$$5 = 5a$$

$$a = 1m^{-2}$$

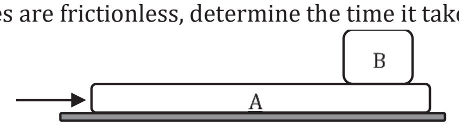
Now, from (1),

$$T = 2(1) = 2\ N$$

Hence, option (B) is correct.



Ex. A small block B is positioned on another block A with a mass of 5 kg and a length of 20 cm. Initially, block B is close to the right end of block A. A consistent horizontal force of 10 N is applied to block A. Assuming all surfaces are frictionless, determine the time it takes for block B to detach from A.



Sol. Every surface is sleek. Block B experiences no horizontal force, resulting in no movement. As block A traverses a distance of 20 cm, block B will detach from it. The free body diagrams (FBD) for both block A and B are illustrated in the diagram.

Acceleration of A,

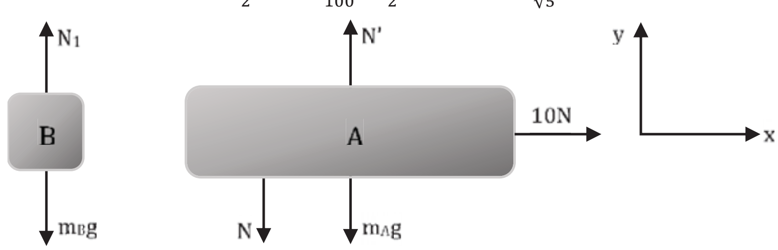
Along x-direction, $\sum \vec{F}_x = m\vec{a}_x$

$$10 = (5)a$$

$$a = 2ms^{-2}$$

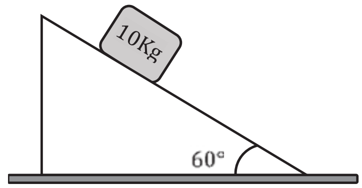
We have, Displacement of the block

$$x = ut + \frac{1}{2}at^2 \Rightarrow \frac{20}{100} = \frac{1}{2}(2)t^2 \Rightarrow t = \frac{1}{\sqrt{5}} = 0.45s$$



Ex. A 10 kg mass is positioned on a wedge fixed to the ground, inclined at an angle of 60° to the horizontal. The body is then released. What is the speed of the block after 2 seconds? (Where $g = 10\ ms^{-2}$)

(A) $10\sqrt{3}ms^{-1}$ (B) $5\sqrt{3}ms^{-1}$ (C) $5ms^{-1}$ (D) $10ms^{-1}$



Sol. The Free Body Diagram (FBD) for the 10 kg block is depicted below.

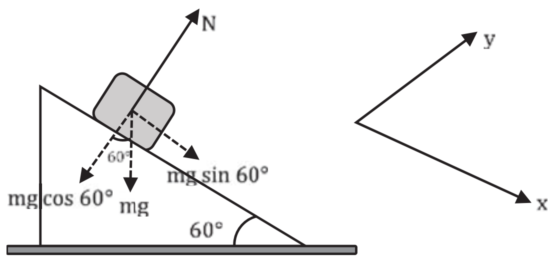
In this representation, the x-axis is aligned along the downward inclined surface to correspond with the direction of the body's acceleration.

$$\sum \vec{F}_x = m\vec{a}_x$$
$$100\sin 60^\circ = (10)a$$
$$50\sqrt{3} = (10)a$$
$$a = 5\sqrt{3}ms^{-2}$$

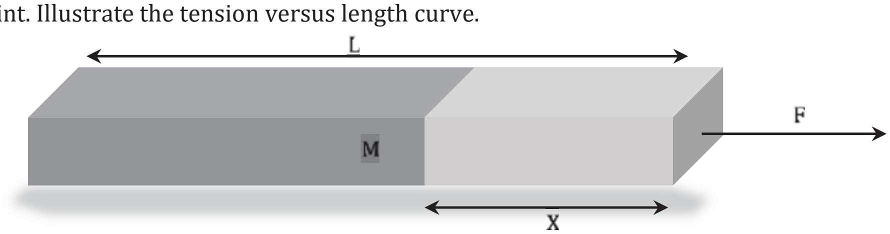
Here, initial velocity, $u = 0$

Hence,
$$V = u + at$$
$$V = 0 + (5\sqrt{3})(2)$$
$$V = 10\sqrt{3}ms^{-1}$$

Hence, option (A) is correct.

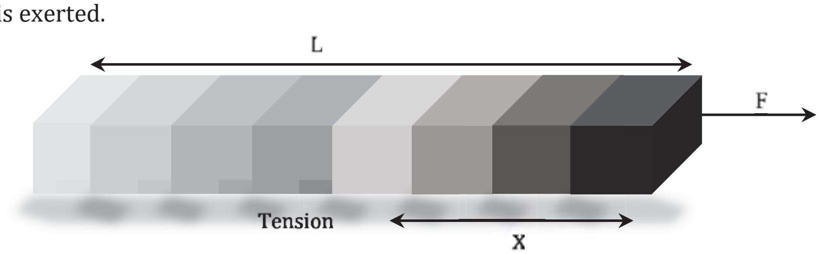


Ex. A uniform rod of length L and mass M is placed on a frictionless surface, and a horizontal force F is exerted at one end. Determine the tension in the rod at a distance x from the force application point. Illustrate the tension versus length curve.



Sol. Greater mass to pull results in higher tension. Consequently, tension will uniformly fluctuate along the length due to the consistent cross-section of the rod, with the highest tension occurring at the initial segment. This is visually represented with a color code, where the red color signifies the maximum tension.

Let's partition the rod into two segments and fracture it at a distance x from the end where the force is exerted.



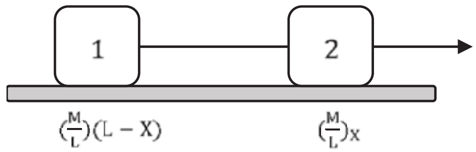
Masses of two sections of rod are:
For section with length x,

$$m_2 = (\frac{M}{L})X$$

For section with length (L − x)

$$m_1 = (\frac{M}{L})(L - X)$$

FBD for the two sections is as follows:



For section with length x,

$$m_2 = (\frac{M}{L})X$$

For section with length (L − x),

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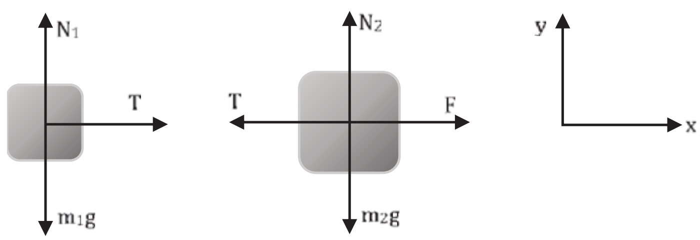
FBD for the two sections is as follows
For body 1,
Along x-direction $T = \frac{M}{L}(L - x)a$ ... (1)
For body 2,
Along x-direction, $F - T = \frac{M}{L}(x)a$ ... (2)
Adding (1) and (2),

$$F = Ma$$



Substitute a in equation (1),

$$T = \frac{M}{L}(L - X)(\frac{F}{M})$$

$$T = \frac{F}{L}(L - X)$$

$$T = F(1 - \frac{X}{L})$$

At $x = 0, T = F,$
At $x = L, T = 0,$
Tension changes in a linear manner with the length of the rod. Therefore, the graph depicting tension versus length can be illustrated as depicted in the adjacent figure.