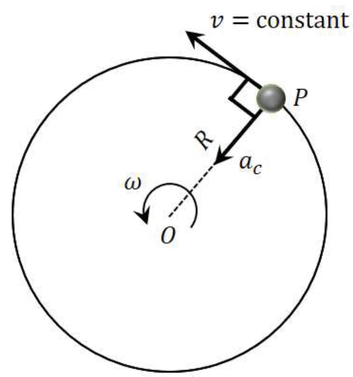
UNIFORM AND NON UNIFORM CIRCULAR MOTION

Uniform Circular Motion

If a particle's speed remains the same, it is described as undergoing Uniform Circular Motion (UCM).

At each moment, the velocity's direction alters, staying tangent to the circular route.



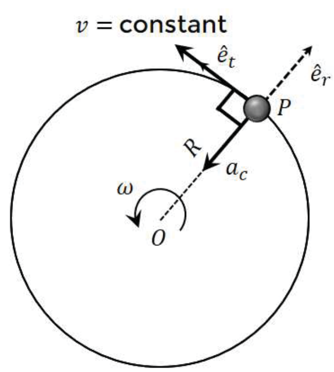
$v = R\omega$

$\omega =$ Constant $\because v =$ Constant

$\alpha = \frac{d\omega}{dt} = 0$

$\alpha =$ Angular acceleration

Characteristics of Uniform Circular Motion



$\omega =$ Constant , $\alpha = 0$

$a_c = \frac{v^2}{R} = \omega^2 R, a_t = 0$

Time period $(T) = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$

Frequency $(f) = \frac{1}{T} = \frac{\omega}{2\pi}$

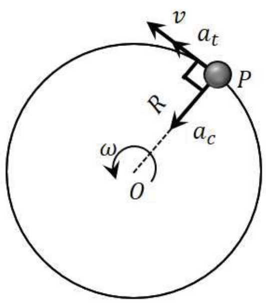
Angle between $\vec{a}_c$ and $\vec{v}$ is 90°

Angle between $\vec{a}_c$ and radial vector $(\hat{e}_r)$ is180°

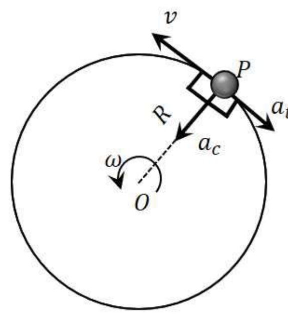
Non-uniform Circular Motion

The particle moves in a circular manner, where both the speed and the direction of its motion constantly shift.

Acceleration consists of two components: one that alters the speed of velocity, known as the tangential component (at), and another that changes the direction of velocity, known as the radial component (ac or ar).

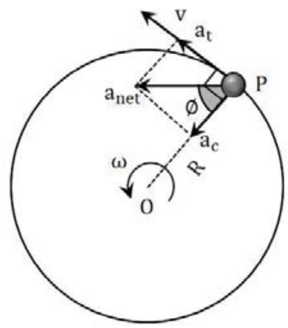


Magnitude of velocity v increases

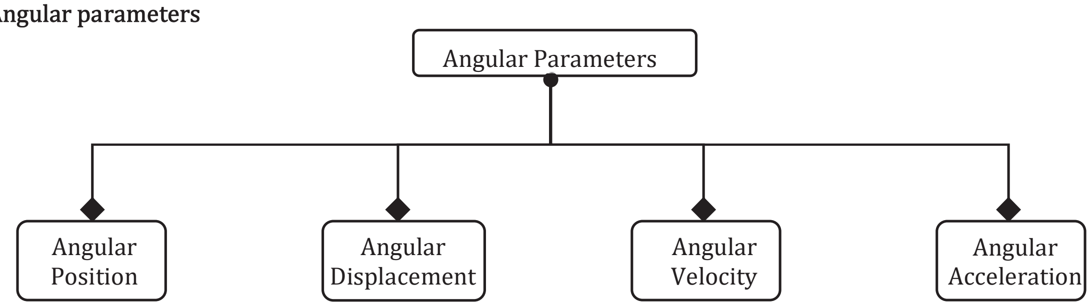


Magnitude of velocity v decreases

$$v = R\omega$$
$$a_t = \frac{dv}{dt} = \frac{Rd\omega}{dt} = R\alpha$$
$$a_c = \frac{v^2}{R} = \omega^2 R$$
$$a_{net} = \sqrt{a_t^2 + a_c^2}$$
$$a_{\text{net}} = \sqrt{a_t^2 + a_c^2}$$

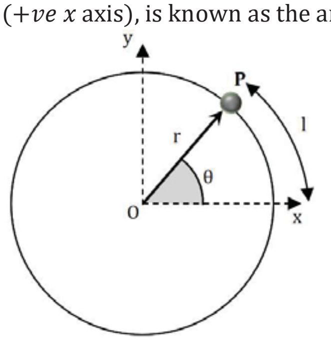


Angular parameters



Angular Position

The angle formed by the line connecting the particle's position to the starting point (origin), measured from the reference line ($+ve\ x$ axis), is known as the angular position (θ).

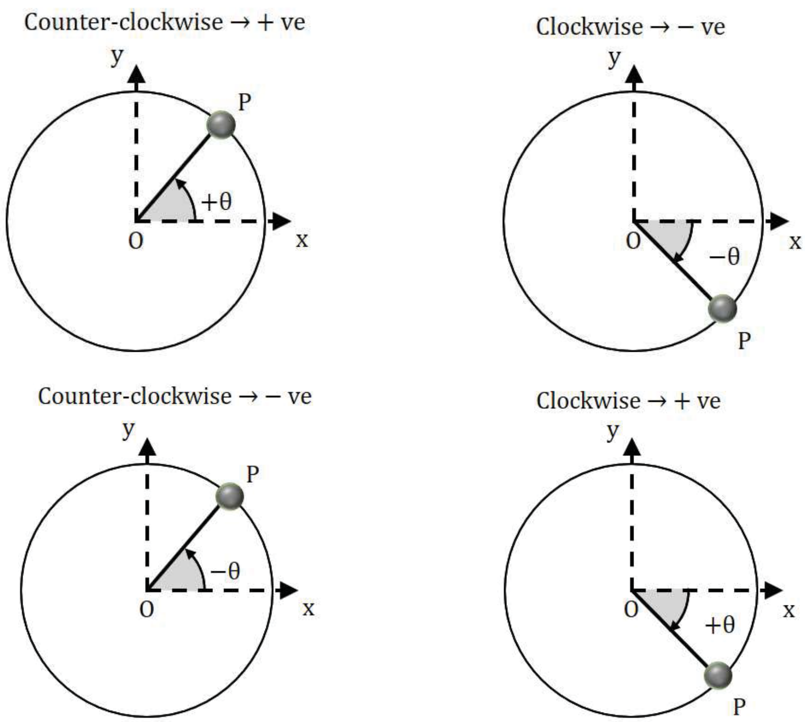


$$\theta = \frac{\text{Arc length}}{\text{Radius}} = \frac{l}{r}$$

In a complete revolution

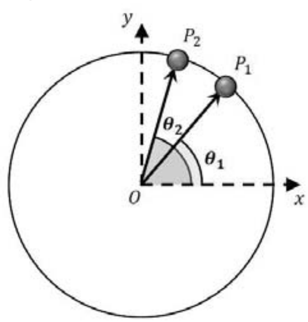
$$\theta = \frac{\text{Arclength}}{\text{Radius}} = \frac{2\pi r}{r} = 2\pi\text{rad}$$
$$1 \text{ revolution } = 2\pi\text{rad} = 360^\circ$$
$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180}\text{rad}$$

Sign convention



Angular Displacement

The Angle ($\Delta\theta$) is the amount by which the position line of the moving object turns during a specific time period concerning a defined origin and axis.

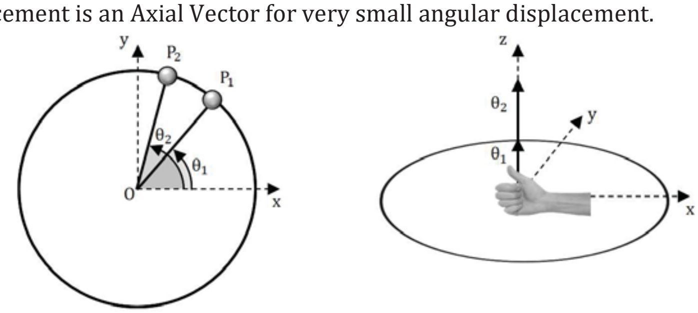


It represents the gap between two angular positions or coordinates.

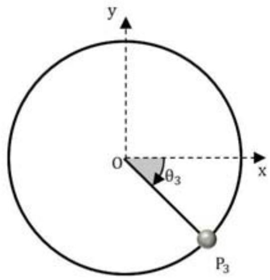
$$\Delta\theta = \theta_2 - \theta_1$$

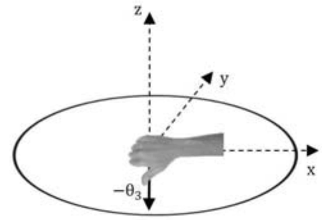
Direction of Angular Displacement

Angular displacement is an Axial Vector for very small angular displacement.



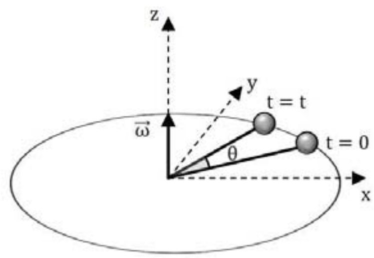
If you curl your right hand's fingers along the direction of the object's rotation, your thumb will indicate the direction of the axial vector for this angular change.





Angular Velocity

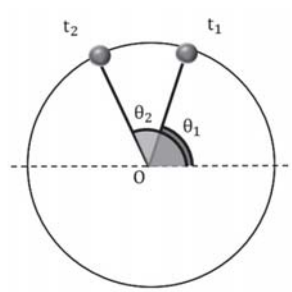
Angular Velocity (ω) is the speed at which the angular position changes over time.
It is an axial vector



SI unit : rad/s

$$\omega = \frac{\theta}{t}$$

Average Angular Velocity



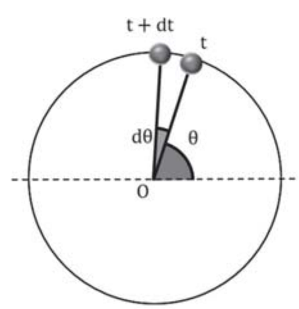
If θ_1 and θ_2 are the angular positions at time t_1 and t_2 respectively, then

$$\text{Average Angular Velocity } = \frac{\text{Angular Displacement}}{\text{Total Time Taken}}$$

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

SI unit of Average Angular Velocity is rad s^{-1}

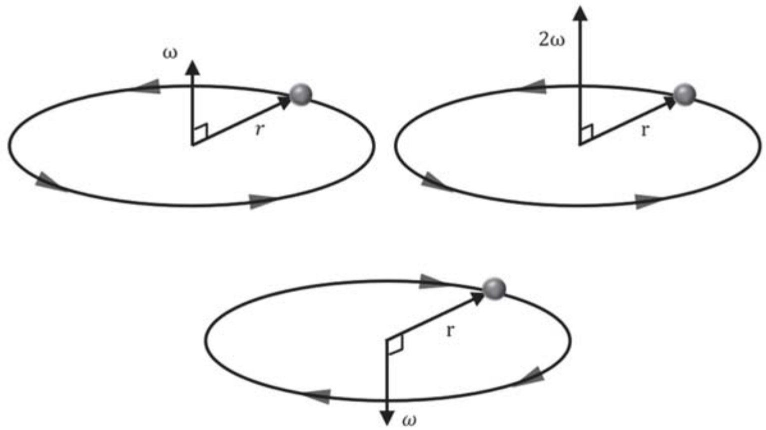
Instantaneous Angular Velocity



Instantaneous Angular Velocity refers to the immediate speed at which the angular position changes at any given moment in time.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Velocity is an axial vector.



Common Units of Angular Velocity

SI unit : rad/s

Revolutions Per Minute (RPM)

$$1RPM = \frac{2\pi}{60}\text{rads}^{-1}$$
$$NRPM = \frac{2\pi N}{60}\text{rads}^{-1}$$

Example- Angular velocity in $RPM = 90$

$$\omega = 3\pi\text{rads}^{-1}$$

Revolutions Per Second (RPS)

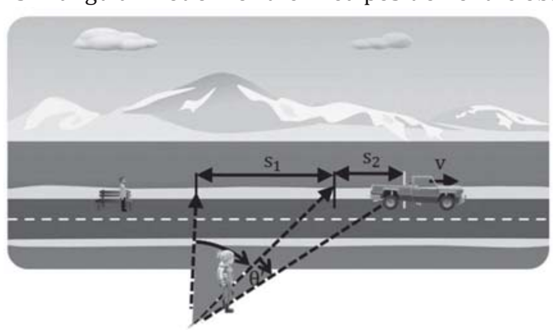
$$1RPS = 2\pi\text{rads}^{-1}$$
$$NRPS = 2\pi N\text{rads}^{-1}$$

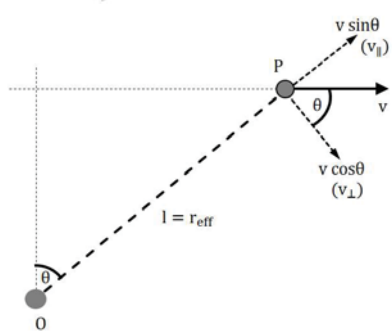
Example- Angular velocity in $RPS = 10$

$$\omega = 20\pi\text{rads}^{-1}$$

Angular Velocity for General Motion

To someone standing on the road, the car appears to move straightforwardly. But for an observer standing at a distance from the road, they need to turn their head to follow the car's motion.
The path of the car is in angular motion for the fixed position of the observer.





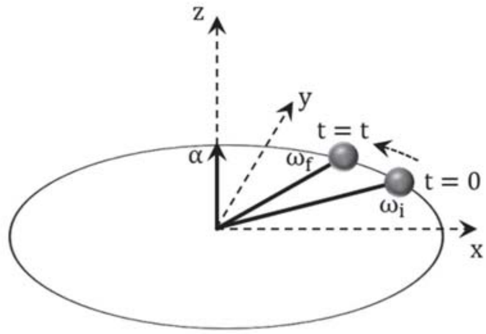
Take parallel and perpendicular components of the linear velocity v.
Consider that the observer is at the centre of a circle whose effective radius is l.
The object is travelling in a circle with a velocity component perpendicular to the effective radius l.
Instantaneous angular velocity for position of particle P about point O;

$$\omega = \frac{v_\perp}{l} = \frac{v\cos\theta}{r_{eff}}$$

$v_\perp$ angular is component of v along l or r_{eff}

Angular Acceleration

The rate of change of angular velocity with respect to time.



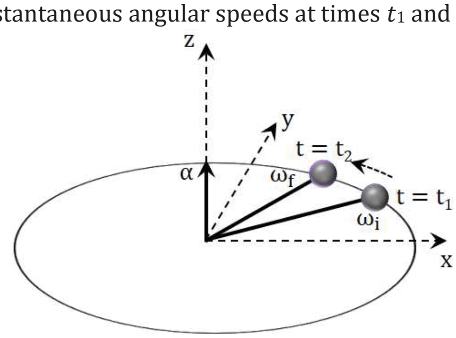
$$\alpha = \frac{\omega_f - \omega_i}{t}$$

SI unit: rad/s^2
It's an axial vector

Types of Angular Acceleration

Average Angular Acceleration

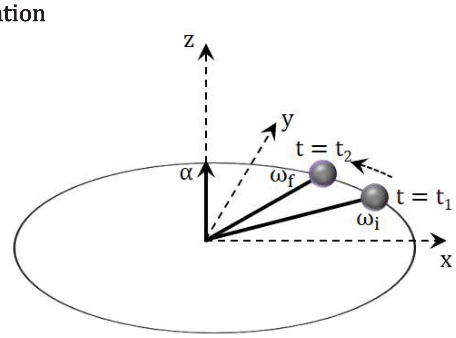
Let ω_i and ω_f be the instantaneous angular speeds at times t_1 and t_2 respectively,



The average angular acceleration,

$$\alpha = \frac{\omega_f - \omega_i}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration



It is the limit of average angular acceleration as Δt approaches zero i.e.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Since

$$\omega = \frac{d\theta}{dt}$$

∴

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Also,

$$\alpha = \omega \frac{d\omega}{d\theta}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$$

Analogy of Linear and Angular Parameters

Linear and angular parameters can be likened to different dimensions of movement, much like the contrast between traveling on a straight road versus navigating through a winding path.

Imagine you're driving a car down a long, straight highway. The linear parameter here would be the distance covered along this road. It's a direct measure of how far you've traveled from your starting point to your current location. Linear parameters deal with straight-line movement, like walking in a straight line or driving on a straight road.

Now, switch gears to a scenario where you're steering a boat through a meandering river. Here, the angular parameter comes into play. Instead of measuring distance along a straight line, it measures the angle or direction of movement. As you navigate bends and turns in the river, the angular parameter tells you how much you've rotated or changed direction from your starting point. Angular parameters deal with rotational movement, like turning a steering wheel or rotating an object.

In essence, linear parameters focus on the straight-ahead progress, while angular parameters track the twists and turns along the way. Both are crucial for understanding motion in different contexts, just like how knowing both the distance traveled and the direction faced are essential for getting from point A to point B, whether you're on a straight highway or a winding river.