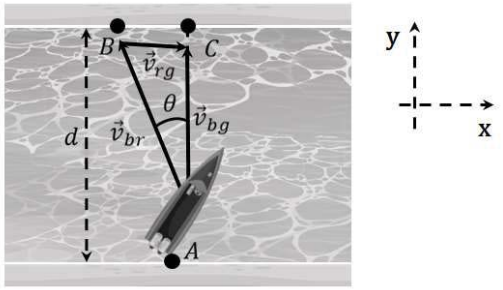
RIVER-MAN PROBLEM AND ITS APPLICATIONS

River- Swimmer Problems

Condition for Minimum Distance

Case I:



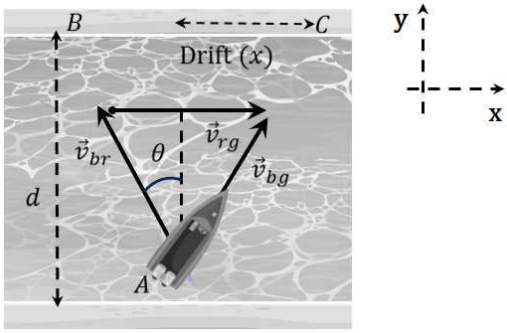
$$|\vec{v}_{br}| > |\vec{v}_{rg}|$$

For zero drift

$$v_{br}\sin\,\theta = v_{rg}$$

$$\sin\,\theta = \frac{v_{rg}}{v_{br}}$$

Case II:

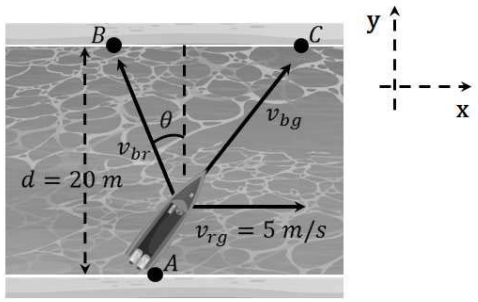


$|\vec{v}_{rg}| > |\vec{v}_{br}|$

$$\theta = \sin^{-1}\left(\frac{v_{br}}{v_{rg}}\right)$$

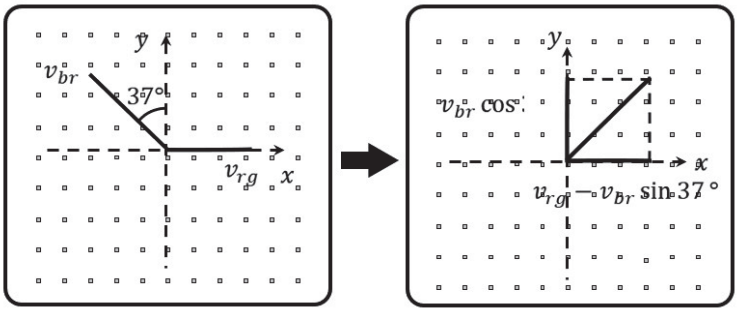
$$\sin\,\theta = \frac{\text{smaller speed}}{\text{larger speed}}$$

Ex A boatman wishes to cross a river along the shortest possible path. The speed of the boat in still water is 3 m/s. The speed of the river is 5 m/s as shown in figure. The width of the river is 20 m. Find the time required to cross the river.



Sol. Given

$$v_{br} = 3\ m/\ ,\ v_{rg} = 5\ m/s$$



To Find: time

Here ; $v_{br} = 3\ m/s$, $v_{rg} = 5\ m/s$

$$\vec{v}_{bg} = \vec{v}_{br} + \vec{v}_{rg}$$

For shortest path:

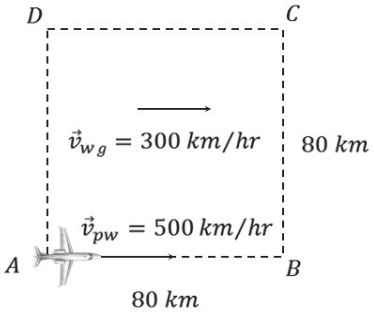
$$\sin\ \theta = \frac{v_{br}}{v_{rg}} \Rightarrow \theta = 37°$$

$$t = \frac{d}{v_{br}\cos\ 37°} = \frac{20}{12/5} = \frac{25}{3}$$

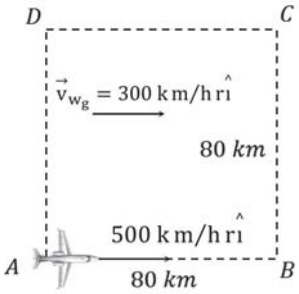
$$t = \frac{25}{3} s$$

Aircraft- wind Problems

Ex. Engine speed of a plane is 500 km/hr, wind is blowing with constant speed of 300 km/hr towards east direction. Find the time taken by plane to complete a square of side 80 km.



Sol. Choose the positive X-axis along the east



Velocity of the plane with respect to the wind is given as :

$$\vec{v}_{pw} = 500 km/hr\ \hat{i}$$

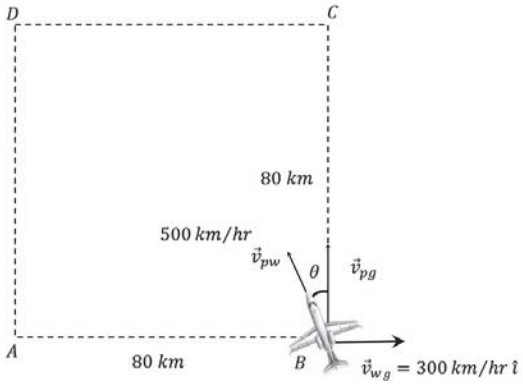
From point A to B

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$$

$$\vec{v}_{pg} = 500\ km/hr\ \hat{i} + 300\ km/hr\ \hat{i} = 800\ km/hr\ \hat{i}$$

$$\text{Time, } t = \frac{80\ km}{800\ km/hr} = 0.1 hr$$

From point *B* to C



$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$$

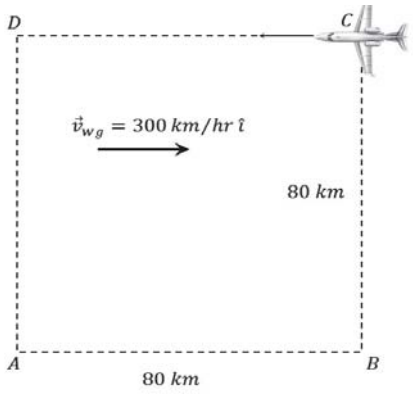
$$\vec{v}_{pg} = \{(300 - 500\sin\theta)\hat{i} + 500\cos\theta\hat{j}\}\text{km/hr}$$

$$300 - 500\sin\theta = 0 \Rightarrow \theta = 37°$$

$$\vec{v}_{pg} = 400\ \text{km/hr}\hat{j}$$

$$\text{Time, t} = \frac{80\ \text{km}}{400\ \text{km/hr}} = 0.2\text{hr}$$

From point *C* to D



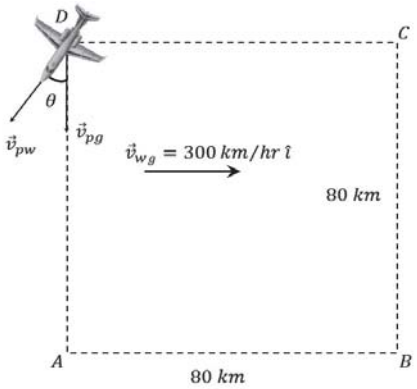
$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$$

$$\vec{v}_{pg} = -500\ \text{km/hr}\hat{i} + 300\ \text{km/hr}\hat{i}$$

$$\vec{v}_{pg} = -200\ \text{km/hr}\hat{i}$$

$$\text{Time, t} = \frac{80\ \text{km}}{200\ \text{km/hr}} = 0.4\text{hr}$$

From point *D* to *A*



$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg}$

$\vec{v}_{pg} = \{(300 - 500\sin\theta)\hat{i} - 500\cos\theta\hat{j}\}\text{km/hr}$

$(v_{pg})_x = 0 \Rightarrow \theta = 37^0$

$\vec{v}_{pg} = -400\ \text{km/hr}\hat{j}$

$$\text{Time} = t = \frac{80\ \text{km}}{400\ \text{km/hr}} = 0.2\text{hr}$$

$\text{Total time} = 0.9\text{hr} = 54\ \text{min}$