CROSS PRODUCT AND INTRODUCTION TO CIRCULAR MOTION

Cross Product of Vectors

The cross product, also known as the vector product, involves the mathematical operation on two vectors to yield a resultant vector, often denoted as $A \times B$. This resultant vector is perpendicular to the plane defined by the two original vectors, A and B. Unlike the dot product, which computes the angle between the vectors, the cross product signifies an orthogonal relationship between the vectors within a three-dimensional space.

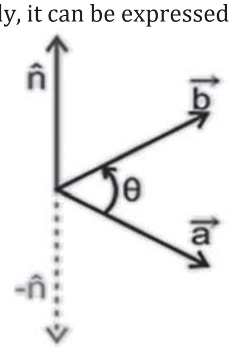
In the context of three-dimensional Euclidean vector space, the cross product is a binary operation performed on two vectors. The procedure for determining the cross product typically employs the right-hand rule, wherein the thumb, index finger, and middle finger of the right hand are positioned to denote the direction of the first vector, the second vector, and the resultant vector, respectively. This rule facilitates the identification of the resultant vector perpendicular to the plane formed by the original vectors.

Furthermore, the magnitude of the resulting vector can be calculated using the cross-product operation. This magnitude represents the product of the magnitudes of the original vectors and the sine of the angle between them. Thus, the cross product serves as a valuable tool for analyzing vectors in three-dimensional space, aiding in various applications across fields such as physics, engineering, and computer graphics.

Cross Product Formula

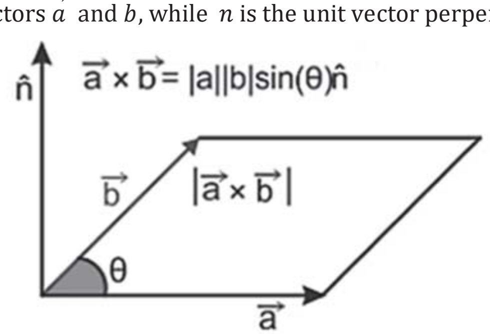
The formula for the cross product between two given vectors yields the area enclosed by those vectors. This mathematical expression represents the magnitude of the resulting vector, which corresponds to the area of the parallelogram formed by the two vectors.

If θ represents the angle between the two given vectors a and b, then the cross product of vectors is denoted as $a \times b$. Mathematically, it can be expressed as:



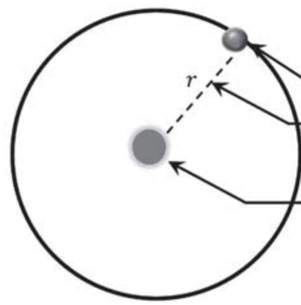
$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$$

Here, $|\vec{a}|$ and $|\vec{b}|$ represent the magnitudes of the vectors $|\vec{a}|$ and $|\vec{b}|$ respectively. The angle θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, while $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$



Introduction to Circular Motion

When an object moves within a flat surface in a way that its distance from a fixed point stays the same, its path is known as a Circle, and this movement is referred to as Circular motion.



Angular parameters of a circular motion

Angular Displacement

The angle which is subtended by the position vector at the center of the circular path refers to the angular displacement.

Angular Displacement $(\Delta\theta) = \left(\frac{\Delta S}{r}\right)$

Where Δ's refers to the linear displacement while r is the radius. Radian is the unit of Angular Displacement.

Angular Acceleration

It refers to the rate of time of change of angular velocity $(d\tilde{\omega})$.

Angular acceleration $(\alpha) = \left(\frac{d\omega}{dt}\right) = (\frac{d^2\theta}{dt^2})$

Its unit is rad/s^2 and dimensional formula $[T]^{-2}$. The relation between linear acceleration (α) and angular acceleration (α)

$A = r\alpha$, where r is the radius.

Angular Velocity

It refers to the time rate change of angular displacement (dω).

Angular Velocity $(\omega) = \frac{\Delta\theta}{\Delta t}$

Angular Velocity is a vector quantity. Its unit is rad/s. The relation between the linear velocity (v) and angular velocity (ω) is

$v = r\omega$

Centripetal Acceleration

It refers to an acceleration that acts on the body in circular motion whose direction is always towards the center of the path

Centripetal Acceleration $a_c = \frac{v^2}{r} = r\omega^2$,

The magnitude of this acceleration by comparing ratios of velocity and position around the circle. Since the particle is traveling in a circular path, the ratio of the change in velocity to velocity will be the same as the ratio of the change in position to position. It is also known as radial acceleration as it acts along the radius of the circle. Centripetal Acceleration is a vector quantity and the unit is in m/s^2.