ANALYSIS OF PROJECTILE ON AN INCLINED PLANE

Elastic collision of Projectile motion

In the realm of physics, a collision denotes an event where two or more bodies interact by exerting forces upon each other over a short span of time. A prime illustration of this concept is evident in traffic collisions, colloquially termed car accidents, wherein a vehicle abruptly encounters another vehicle, object, or individual.

These collisions are typically classified into two main categories:

(i) inelastic

(ii) elastic

An instance of an inelastic collision occurs when two cars collide, resulting in a dissipation of kinetic energy due to internal friction. Conversely, an elastic collision, such as when two balls collide during a game of pool, upholds the conservation of both momentum and kinetic energy. This differentiation between the two types of collisions holds significant importance and is grounded in the principles of conservation laws, notably the conservation of momentum and kinetic energy.

In an inelastic collision, although momentum remains constant, kinetic energy undergoes a transformation. This implies that the combined momentum of the objects involved remains unaltered before and after the collision, yet a portion of the kinetic energy converts into alternate forms, such as heat or sound.

For instance, during a traffic collision, a portion of the kinetic energy is dissipated as heat or sound upon impact. Conversely, in an elastic collision, momentum and the majority of kinetic energy remain conserved, with no net conversion of kinetic energy into alternative forms. In instances where all kinetic energy is preserved, it is termed a perfectly elastic collision. However, such occurrences are rare, as collisions typically entail some degree of kinetic energy dissipation, as evidenced by the conversion of kinetic energy into sound or friction, as observed in the scenario of billiard balls colliding

Here are the detailed characteristics of an elastic collision:

Conservation of Linear Momentum

In an elastic collision, the linear momentum of each object involved remains conserved throughout the interaction. This means that the total momentum before the collision is equal to the total momentum after the collision.

Conservation of Total Energy

The total energy of the objects involved in an elastic collision remains constant. This encompasses both kinetic and potential energy, ensuring that there is no net gain or loss of energy during the collision.

Conservation of Kinetic Energy

In addition to the total energy, the kinetic energy of the system is also conserved during an elastic collision. This implies that the sum of the kinetic energies of the objects before the collision equals the sum of the kinetic energies after the collision.

Involvement of Conservative Forces

Elastic collisions are characterized by the involvement of conservative forces. These are forces that do not dissipate energy as heat or other non-mechanical forms during the collision.

Absence of Energy Conversion into Heat

One of the distinctive features of an elastic collision is that the mechanical energy within the system is not converted into heat. This means that there is no loss of energy due to friction or other dissipative processes, resulting in a purely mechanical interaction.

Projection on inclined plane

A projectile is thrown on an inclined plane of inclination a with initial speed u at an angle ß with the incline.

Analysis of Motion

Let us assume the direction along inclined plane as x-axis and perpendicular to it as y-axis. We have

$$u_x = u\cos\ \beta;\ a_x = -g\sin\ \alpha$$
$$u_y = u\sin\ \beta;\ a_y = -g\cos\ \alpha$$

The accelerations a_x and a_y are constant. The motion along x-axis as well as along y-axis are uniformly accelerated.

Equations for x-axis

Velocity along x-axis after time t, using v = u + at

$$v_x = u\cos\beta + (-g\sin\alpha)t \qquad \dots (1)$$

Displacement along x-axis after time t, using

$$s = ut + \frac{1}{2}at^2$$
$$x = (u\cos\beta)t + \frac{1}{2}(-g\sin\alpha)t^2 \qquad \dots (2)$$

Equations for y-axis

$$v_y = u\sin\ \beta + (-g\cos\ \alpha)t, \text{ and} \qquad \dots (3)$$
$$y = (u\sin\beta)t + \frac{1}{2}(-g\cos\alpha)t^2 \qquad \dots (4)$$

Time of Flight

When the projectile lands at point Q on the inclined at time t = T, y = 0.

$$0 = (u\sin\ \beta)T + \frac{1}{2}(-g\cos\ \alpha)T^2$$
$$T = \frac{2u\sin\ \beta}{g\cos\ \alpha}$$

Range along the incline

In time T, the displacement along x-axis is called range along the incline. Using the equation of motion along x-axis,

$$R = (u\cos\ \beta)T - \frac{1}{2}g\sin\ \alpha T^2 = (u\cos\beta)\left(\frac{2u\sin\beta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\beta}{g\cos\alpha}\right)^2$$
$$R = \frac{2u^2\sin\ \beta\cos\ (\alpha+\beta)}{g\cos^2\ \alpha}$$

Condition for maximum range

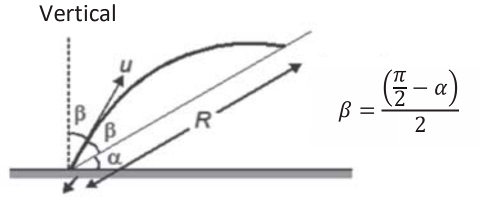
Since R depends on the angle of projection β above the incline plane comma therefore for maximum range $\frac{dR}{d\beta} = 0$;

$$\frac{dR}{d\beta} = \frac{2u^2}{g\cos^2\ \alpha}\frac{d}{d\beta}[\sin\ \beta\cos\ (\alpha+\beta)]$$
$$\frac{2u^2}{g\cos^2\alpha}\cos(\alpha + 2\beta) = 0$$

[∵ $\cos\ (A + B) = \cos\ A\cos\ B - \sin\ A \cdot \sin\ B$]

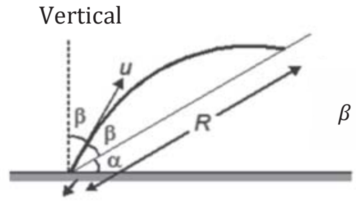
$$\cos\ (\alpha + 2\beta) = 0$$
$$\alpha + 2\beta = \frac{\pi}{2} \text{ or } \beta = \frac{\pi}{4} - \frac{\alpha}{2}$$

Based on the previous discussion, it's evident that for maximum range along the plane, the object should be projected along the line that splits the angle evenly between the vertical (with respect to the ground) and the inclined plane.



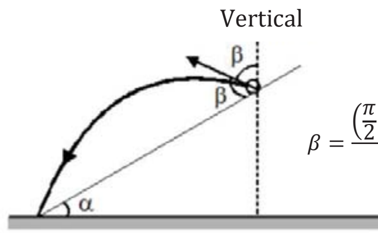
Condition for maximum range for projectile thrown up the plane.

This condition is universally true whether the projectile is projected up the inclined plane, down the inclined or even for projectile on horizontal plane where the range is maximum when angle of projection is maximum for $\left(\frac{\pi}{4} = 45^\circ\right)$ which is the angular bisector of horizontal and vertical planes.



$$\beta = \frac{\left(\frac{\pi}{2} - \alpha\right)}{2}$$

Condition for maximum range for projectile thrown down the plane.



$$\beta = \frac{\left(\frac{\pi}{2} - \alpha\right)}{2}$$

Maximum Range R_{max}

If projectile is thrown up the plane

$$\beta = \frac{\pi}{4} - \frac{\alpha}{2}$$

$$R_{max} = \frac{2u^2 \sin\beta \cos(\alpha+\beta)}{g\cos^2\alpha} = \frac{2u^2\sin^2\beta}{g\cos^2\alpha} = \frac{u^2(1-\cos 2\beta)}{g\cos^2\alpha} = \frac{u^2(1-\sin\alpha)}{g(1-\sin^2\alpha)}$$

$$R_{max} = \frac{u^2}{g(1+\sin\alpha)}$$

If the projectile is thrown down the plane

$$\beta = \frac{\pi}{4} + \frac{\alpha}{2}$$

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

Condition for object to hit the incline perpendicular to it

When the projectile hits the plane perpendicularly, its velocity along the incline (v_x) becomes zero.

Therefore,

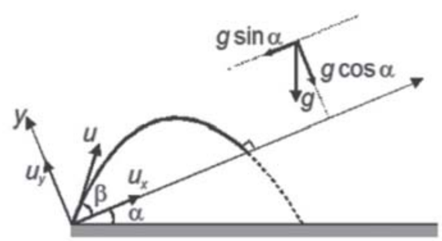
$$v_x = 0 = u\cos\beta - (g\sin\alpha)t$$

$$t = \frac{u\cos\beta}{g\sin\alpha}$$

Equating this with the time-of-flight T, we get

$$\frac{u\cos\beta}{q\sin\alpha} = \frac{2u\sin\beta}{q\cos\alpha}$$

$\tan\alpha \tan\beta = \frac{1}{2}$ is the desired condition.



$$u_x = u\cos\beta$$
$$u_y = u\sin\beta$$