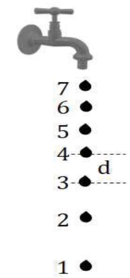
MOTION UNDER GRAVITY

Motion under Gravity

Whenever an object is tossed upward, downward, or let go from a height, it descends freely due to Earth's gravitational pull.



$$v = u + gt$$

$$h = h_0 + ut + \frac{1}{2}gt^2 \text{ or } h - h_0 = s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2g(h - h_0) \text{ or } v^2 = u^2 + 2gs$$

$$h_{n^{th}} = u + \frac{g}{2}(2n - 1)$$

Where h = vertical displacement, $h_{n^{th}}$ = vertical displacement in n^{th} second

Assumptions for Motion under Gravity

These assumptions help in constructing mathematical descriptions of motion that are both accurate and manageable.

Here are some common assumptions:

Uniform Gravitational Field:

One common assumption is that the gravitational field strength is uniform near the surface of the Earth. This means that the acceleration due to gravity (usually denoted as g) is constant regardless of an object's mass or position. In reality, the gravitational field strength does vary slightly with altitude and location, but for most practical purposes, it can be treated as uniform over small distances.

Negligible Air Resistance:

Another assumption is that the effects of air resistance (drag) on the motion of an object are negligible. Air resistance can significantly affect the motion of objects moving through the Earth's atmosphere, particularly at high speeds or for objects with large surface areas. However, in many situations, such as with small or dense objects or over short distances, air resistance can be ignored.

Vertical Motion Only:

When analyzing motion under gravity, it's often assumed that the motion is vertical, meaning it occurs in a straight line along the direction of the gravitational force. This simplifies the analysis and allows for the use of one-dimensional equations of motion.

Point Mass:

For simplicity, objects are often treated as point masses, meaning that their size and shape are ignored, and all of their mass is concentrated at a single point. This assumption is valid when the size of the object is much smaller than the distances over which it moves or the objects with which it interacts.

Constant Gravitational Acceleration:

It's assumed that the acceleration due to gravity (g) remains constant throughout the motion. This allows for the use of constant acceleration equations, such as those derived from Newton's laws of motion.

These assumptions provide a framework for analyzing motion under gravity in various scenarios, such as free fall, projectile motion, and motion along inclined planes. While they simplify calculations, it's important to recognize their limitations and consider more complex factors when necessary, such as variations in gravitational field strength or the effects of air resistance in certain situations.

Numerical on Motion under Gravity

Ex. Water drops fall from a tap at an interval of 1s. What is the distance between the 3rd and 4th drop when the 7th drop just leaves?

Sol. The distance of the 3rd drops from the tap at the 4th second is

$$H_3 = \frac{t^2 g}{2} = \frac{4^2 \times 10}{2} = 80 \text{ m}$$

The distance of the 4th drops from the tap at the 3rd second is,

$$H_4 = \frac{t^2 g}{2} = \frac{3^2 \times 10}{2} = 45 \text{ m}$$

The distance between the 3rd and 4th drops when the 7th drop just leaves is,

$$d = H_3 - H_4 = 80 - 45\text{m}$$

$$d = 35 \text{ m}$$

Average speed and Average Velocity

1. If a body covers s_1 distance with speed v_1 , s_2 with speed v_2 , then its average speed is

$$v_{av} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{\sum s}{\sum \frac{s}{v}}$$

2. If a body coves first half distance with speed v_1 and next half with speed v_2, then

$$\text{Average speed } = \frac{2v_1 v_2}{v_1 + v_2} \text{ (Harmonic mean)}$$

3. If a body travels with uniform speed v_1 for time t_1 and with uniform speed v_2 for time t_2 , then average speed

$$\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{\sum vt}{\sum t}$$

$$\text{If } t_1 = t_2 = \frac{T}{2} \text{ then } v_{av} = \frac{v_1 + V_2}{2} [T = \text{ time of journey}] \text{ (Arithmatic mean)}$$

4. If body covers first one third with speed v_1, next one third with speed v_2 and remaining one third with speed v_3 then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

5. If a body moves from one point (A) to another point (B) with speed v_1 and returns back (from B to A) with speed v_2 then average velocity is 0 but average speed

$$\frac{2v_1 v_2}{v_1 + v_2}.$$