EQUATIONS OF MOTION AND PROBLEMS INVOLVING GRAPHICAL AND CALCULUS APPROACH

Equations of Motion

General equations of motion:

$$v = \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt$$
$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt$$
$$a = \frac{vdv}{dx} \Rightarrow vdv = adx \Rightarrow \int vdv = \int adx$$

Equations of motion of a particle moving with uniform acceleration in straight line:

$$v = u + at$$
$$S = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2aS$$
$$x = x_0 + ut + \frac{1}{2}at^2$$

Here

u = velocity of particle at t = 0

S = Displacement of particle between 0 to t

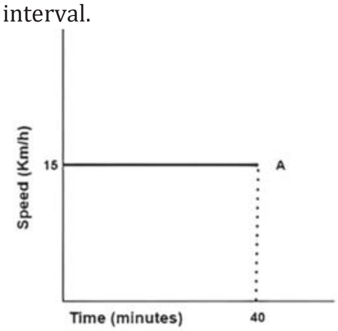
= $x - x_0$ (x_0 = position of particle at t = 0, x = position of particle at time t)

a = uniform acceleration

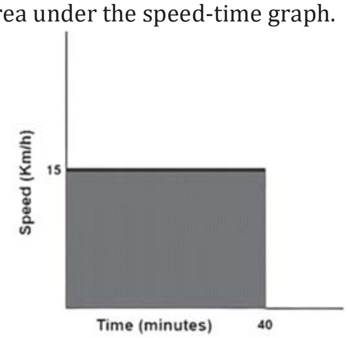
v = velocity of particle at time t

Problems involving graphical

Ex. The graph represents the speed-time relationship of a particle. Determine the distance traveled by the particle within a 40-minute interval.



Sol. Calculating distance using the area under the speed-time graph.



Distance covered by the particle in 40 min = Area under the given curve.

The shape formed under the curve is of a rectangle therefore, the area under the curve = length × width.

Here, the length of the shape is equivalent to the time taken i.e. 40 min or 40 min × 60 s/1 min = 2400 s.

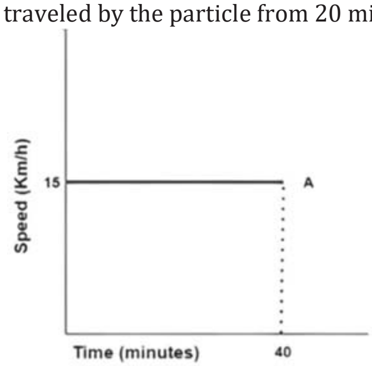
Similarly, the width of the shape is equivalent to the speed i.e. 15 km/h or 15 km/h × 1 h/3600 s × 1000 m/1 km = 4.17 m/s.

Therefore, the area under the curve or the distance covered in 40 min is given by:

2400 s × 4.17 m/s = 10000 m or 10 km.

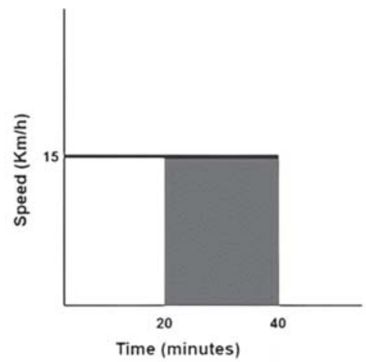
Hence, the distance covered by the particle in 40 min is equal to 10 km.

Ex. The graph depicts the distance traveled by the particle from 20 minutes to 40 minutes.



Sol. Distance covered by the particle from time t = 20 min to 40 min is equal to the Area under the given curve between the given time range.

The shape formed under the curve is of a rectangle therefore, the area under the curve = length × width.



Here, the length of the shape is equivalent to the time taken i.e. (40 – 20) min or 20 min × 60 s/1 min = 1200 s.

Similarly, the width of the shape is equivalent to the speed i.e. 15 km/h or 15 km/h × 1 h/3600 s × 1000 m/1 km = 4.17 m/s.

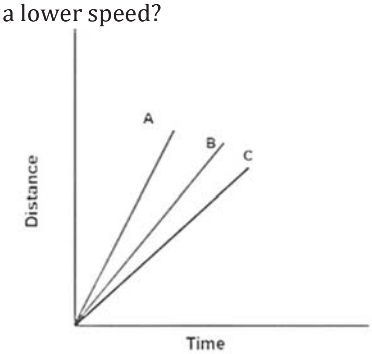
Therefore, the area under the curve or the distance covered from time t = 20 min to 40 min is given by: 1200 s × 4.17 m/s = 5004 m or 5.004 km.

Hence, the distance covered by the particle from time t = 20 min to 40 min is equal to 5.004 km.

Ex. The diagram below illustrates the distance-time graphs of three objects labeled A, B, and C. Identify:

(a) Which object is exhibiting a higher speed?

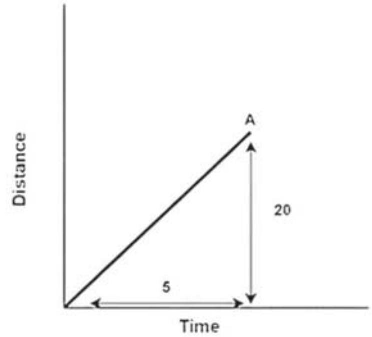
(b) Which object is displaying a lower speed?



Sol. The provided graph represents a distance-time relationship, with the slope indicating the speed, where a steeper slope corresponds to a higher speed.

From the graph analysis, it's evident that object A exhibits the greatest slope, indicating the highest speed, while object C displays the shallowest slope, representing the slowest speed.

Ex. Determine the velocity of the particle using the provided distance-time graph of its motion.



Sol. Here, the speed of the particle is equal to the slope of the graph.

And the slope of the distance graph is equal to the speed of the particle.

Therefore, the formula to calculate the speed of the particle is:

$$\text{Speed} = \text{Distance} / \text{Time}$$
$$= 20 \text{ m} / 5 \text{ s} = 4 \text{ m/s}.$$

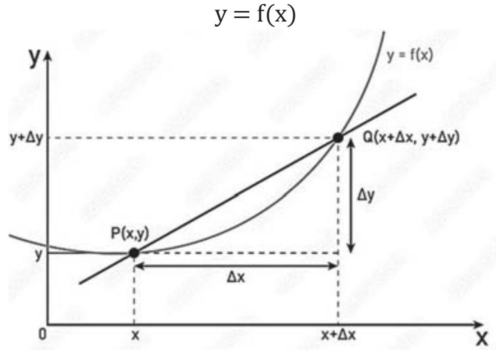
Hence, the speed of the particle is equal to 4 m/s.

Calculus approach

Using calculus, we can derive equations of motion to determine velocity and acceleration, which are crucial for understanding physical quantities involved in motion.

Let us make an equation using a quantity (y) and a single variable (x). it can be written as:

$$y = f(x)$$



P and Q are joined by a slope, and the equation of slope is:

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{(y+\Delta y)-y}{\Delta x}$$

This is the equation by differential calculus.