Chapter 3

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VECTORS AND VECTOR ADDITION

Vectors & Scalars Representation of Vectors Types of Vectors Angle Between Vectors

Scalar and Vector Quantities are employed to depict how an object moves. Scalar Quantities are those which solely possess size or magnitude, such as distance, speed, mass, and density. Conversely, vector quantities include both magnitude and direction, like displacement, velocity, acceleration, and force. It's worth mentioning that when a vector quantity alters, both its magnitude and direction alter accordingly, whereas when a scalar quantity alters, only its magnitude changes.

Scalar quantities

Scalar quantities are frequently compared to vector quantities, which include both size and direction, like velocity, acceleration, force, and displacement. Vector quantities are usually depicted with arrows to indicate both their size and direction, while scalar quantities are shown simply with a number and a unit.

There are various types of scalar quantities, some of which are outlined below:

Mass, Speed, Distance, Time, Area, Volume, Density, Energy, Temperature, Electric Charge, Gravitational force

Vector Quantities

Vector quantities find applications across various scientific and engineering disciplines like mechanics, electromagnetism, fluid dynamics, and quantum mechanics. They play a crucial role in explaining how physical systems behave and in predicting what will happen next.

In everyday life, we encounter numerous examples of vector quantities. Here are a few examples listed below!

Velocity, Force, Pressure, Displacement, Acceleration, Thrust, Linear momentum, Electric field, Polarization, Weight

Vector representation

Vectors are usually shown using symbols that have arrows on top of them. This helps to differentiate them from scalar quantities.

$$\vec{AB}, \vec{BC}$$

Alternatively, individual letters can also stand for vectors, like this:

$$\text{Velocity } = \vec{v}, \text{Force } = \vec{F}$$

The arrow notation serves as a visual cue, indicating that a quantity has both size and direction. Vectors can also be shown graphically as arrows in diagrams or coordinate systems. Here, the length of the arrow shows the vector's size, while the arrow's direction indicates its orientation.

Types of Vectors

Equality of vectors

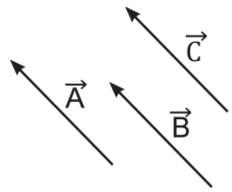
Vectors that have the same size and point in the same direction are called equal vectors.

If

$$|\vec{A}| = |\vec{B}| = |\vec{C}| \text{ and } \hat{A} = \hat{B} = \hat{C}$$

Then

$$\vec{A} = \vec{B} = \vec{C}$$



Null vector

The magnitude of the null vector is zero and its direction is random.

Collinear

Vectors on the same line are also known as collinear vectors.

If two vectors are collinear, then one can be expressed in terms of the other

$\vec{A}$ and $\vec{B}$ are collinear vectors because they are on the same line and $\vec{A} = \lambda\vec{B}$

λ is constant.

Both vectors are collinear. Therefore, their directions are the same.

$$\hat{A} = \hat{B}$$

$$\frac{\vec{A}}{|\vec{A}|} = \frac{\vec{B}}{|\vec{B}|}$$

$$\vec{A} = \frac{|\vec{A}|}{|\vec{B}|}\vec{B}$$

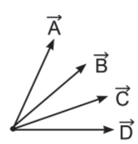
$\frac{|\vec{A}|}{|\vec{B}|}$ is constant, therefore,

$$\vec{A} = \lambda\vec{B}$$

Co-initial vectors

When two or more vectors begin at the same point, they are called co-initial vectors.

Here $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are co-initial vectors.



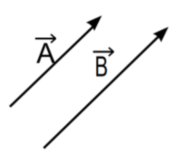
Parallel vectors

If the angle between any two vectors is zero then they are known as parallel vectors.

$$\theta = 0^\circ$$

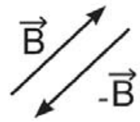
$$|\vec{A}| \neq |\vec{B}|$$

$$\vec{A} \parallel \vec{B}$$



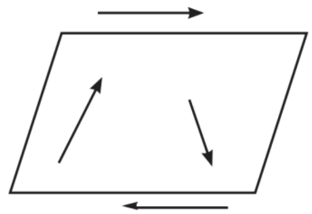
Negative vector

These vectors are the same in magnitude but opposite in direction.



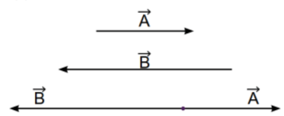
Coplanar vectors

When three or more vectors are in the same plane or parallel to it, they're called coplanar vectors. Two free vectors are always coplanar.



Anti-parallel vectors

These vectors have different magnitudes and are opposite in direction.



Angle Between Vectors

If we consider the two vectors to be $\vec{a}$ and $\vec{b}$ then the dot created is articulated as $\vec{a} \cdot \vec{b}$
Suppose these two vectors are separated by an angle θ.
To find out the angle's measurement, we use the formula below:
we know that the dot product of two product is given as

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

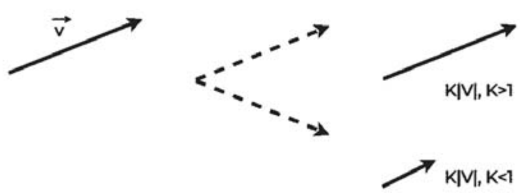
where θ is the angle between $\vec{a}$ and $\vec{b}$

Multiplication of Vector by Scalar

When you multiply a vector a by a constant number k, the resulting vector keeps pointing in the same direction but its size changes by a factor of k. The diagram illustrates the vector before and after it's multiplied by k. Mathematically, this is expressed as:

$$|k\vec{v}| = k|\vec{v}|$$

if $k > 1$, the magnitude of the vector increase while it decreases when the $k < 1$.



Addition of Vectors by Triangular Law and By
Triangle law of vector addition

The Triangle Law of vector addition states that if you represent two vectors with two sides of a triangle, considering their size and direction in the same order, then the third side of that triangle, taken in the opposite order, represents both in size and direction the combined effect of the two vectors.

$\vec{R} = \vec{A} + \vec{B}$

$|\vec{A}| = A; |\vec{B}| = B$

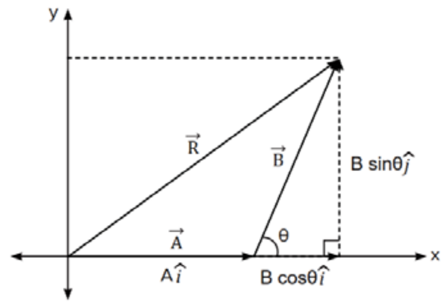
$\vec{A} = A\hat{i}$

$\vec{B} = B\cos\theta\hat{i} + B\sin\theta\hat{j}$

$\vec{R} = (A + B\cos\theta)\hat{i} + B\sin\theta\hat{j}$

$|\vec{R}| = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2\ A\ B\cos\theta}$



Parallelogram law of vector addition

When two vectors, starting from the same point, depict two neighboring sides of a parallelogram, considering their size and direction, the combined vector is shown both in size and direction by the diagonal passing through that starting point.

$\vec{R} = \vec{A} + \vec{B}$

$|\vec{A}| = A; |\vec{B}| = B$

$\vec{A} = A\hat{i}$

$\vec{B} = B\cos\theta\hat{i} + B\sin\theta\hat{j}$

$\vec{R} = (A + B\cos\theta)\hat{i} + B\sin\theta\hat{j}$

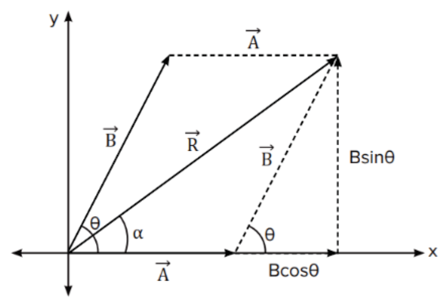
$|\vec{R}| = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\alpha =$ Angle between $\vec{R}$ and $\vec{A}$

$\alpha = \tan^{-1}(\frac{B\sin\theta}{A + B\cos\theta})$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$



Law of Cosines Applications of Law of Cosines

The law of cosines establishes a connection between the lengths of a triangle's sides and the cosine of its angles. This principle is alternatively known as the cosine rule. In the context of a triangle ABC, the statement of the cosine law states:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$, where a, b, and c represent the sides of the triangle, with 'a' being the angle formed between sides 'b' and 'c'.

Likewise, if β and γ represent the angles formed between sides 'ca' and 'ab', respectively, then in accordance with the law of cosines, we obtain:

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = b^2 + a^2 - 2ab\cos\gamma$$

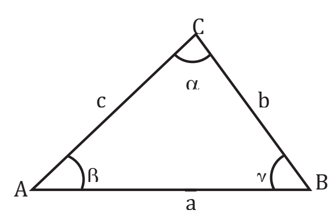
Observation: If any of the angles, α, β, or γ, equals 90 degrees, then the preceding expression will validate the Pythagorean theorem, since cos 90 equals 0. Therefore, the aforementioned three equations can be articulated as:

$$a^2 = b^2 + c^2 [\text{if } \alpha = 90 \text{ degrees}]$$
$$b^2 = a^2 + c^2 [\text{ if } \beta = 90 \text{ degrees}]$$
$$c^2 = b^2 + a^2 [\text{ if y} = 90 \text{ degrees}]$$

Law of Cosines Definition

In Trigonometry, the law of Cosines, alternatively called the Cosine Rule or Cosine Formula, establishes a connection between the sides of a triangle and the cosine of one of its angles. It asserts that if we know the lengths of two sides and the angle between them in a triangle, we can calculate the length of the third side. This relationship is expressed as:

$c^2 = a^2 + b^2 - 2ab\cos\gamma$



Here, a, b, and c represent the sides of a triangle, with γ denoting the angle between sides a and b. Refer to the figure below for visual representation.

Formula

According to the cosine law formula, when determining the lengths of the sides of a triangle such as △ABC, we can express it as:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = b^2 + a^2 - 2ba\cos\gamma$$

If our aim is to ascertain the angles of △ABC, we utilize the cosine rule in the following manner:

$$\cos\alpha = \frac{[b^2+c^2-a^2]}{2bc}$$
$$\cos\beta = \frac{[a^2+c^2-b^2]}{2ac}$$
$$\cos\gamma = \frac{[b^2+a^2-c^2]}{2ab}$$

Here, a, b, and c represent the lengths of the sides of a triangle.

Solving SSS Congruency

In SSS congruence, when the lengths of all three sides of a triangle are known, and the measurement of an unknown angle is needed, the law of cosines can be employed to determine the missing angle.

Initially, we must determine one angle utilizing the cosine law, let's say

$$\cos\alpha = \frac{[b^2+c^2-a^2]}{2bc}.$$

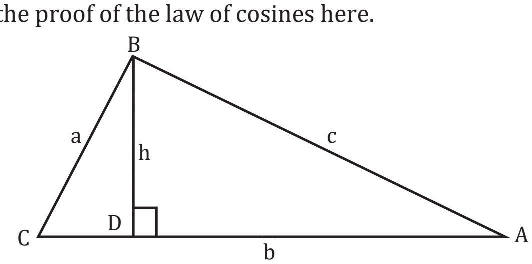
Subsequently, we will calculate the second angle once more using the same law.

$$\cos\beta = \frac{[a^2+c^2-b^2]}{2ac}$$

Now, the third angle can be determined simply by applying the angle sum property of a triangle. This property states that the sum of all three angles in a triangle equals 180 degrees.

Proof

Now, let's explore the proof of the law of cosines here.



In the right triangle BCD, according to the definition of the cosine function

$$\cos C = \frac{CD}{a}$$
$$CD = a\cos C \qquad \dots (1)$$

Subtracting above equation from side b, we get

$$DA = b - a\cos C$$

In the triangle BCD, according to Sine definition

$$\sin C = \frac{BD}{a}$$
$$BD = a\sin C \qquad \dots (2)$$

When applying the Pythagorean Theorem in triangle ADB, then

$$c^2 = BD^2 + DDA^2$$

Substituting for BD and DA from equations (1) and (2)

$$c^2 = (a\sin C)^2 + (b - a\cos C)^2$$

By Cross Multiplication we get:

$$c^2 = a^2\sin^2 c + b^2 - 2ab\cos C + a^2\cos^2 C$$

Rearranging the above equation:

$$c^2 = a^2\sin^2 c + a^2\cos^2 c + b^2 - 2ab\cos C$$

Taking out a^2 as a common factor, we get;

$$c^2 = a^2(\sin^2 c + \cos^2 c) + b^2 - 2ab\cos C$$

Now from the above equation, you know that,

$$\sin^2\theta + \cos^2\theta = 1$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Hence, the cosine law is proved.