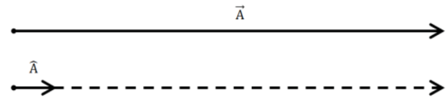
RESOLUTION OF VECTORS

Unit Vector in Cartesian

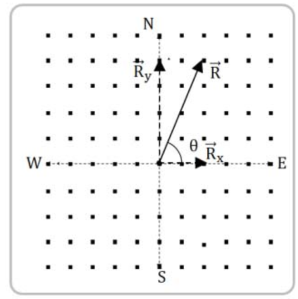
Vector divided by its own magnitude is a vector with unit magnitude and in direction along the parent vector.



$$\frac{\vec{A}}{|\vec{A}|} = \hat{A}$$

$$\vec{A} = |\vec{A}|\hat{A}$$

Resolution of a Vector



$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R}_x = R_x\hat{\imath} \text{ and } \vec{R}_y = R_y\hat{\jmath}$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

$$R_x = R\cos\theta \text{ and } R_y = R\sin\theta$$

Parallelogram Quantitative Approach Vector Subtraction

Vector Subtraction

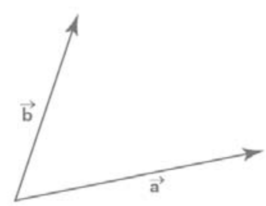
Vector subtraction operates similarly to scalar subtraction, where we subtract the corresponding components of vectors. The graphical representation of vector subtraction can be comprehended through the parallelogram law and triangle law of vector addition.

Vector subtraction, denoted as a - b, entails adding the negative of vector b to vector a, expressed as a - b = a + (-b). Therefore, vector subtraction involves vector addition and the negation of a vector. The outcome of vector subtraction remains a vector. The subsequent rules delineate the process of subtracting vectors:

Vector subtraction should exclusively involve two vectors, not a vector and a scalar.

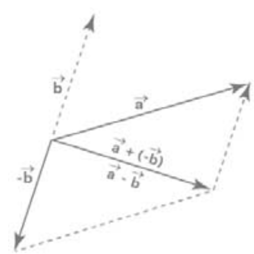
Both vectors participating in the subtraction must signify the same physical quantity.

Vector Subtraction by Parallelogram Law



If a and b are two vectors, how do we graphically interpret the subtraction of these vectors? What significance does a - b hold? Initially, we recognize that a - b results in a vector that, when added to b, yields a, i.e.$(a - b) + b = a$

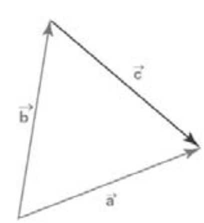
However, how do we ascertain the vector a - b, when provided with the vectors a and b? The illustration below displays vectors a and b (depicted as originating from the same point).



Using the parallelogram law of vector addition, we can determine the vector as follows. We interpret a - b as a + (-b), that is, the vector sum of a and –b. Now, we reverse vector b, and then add a and -b using the parallelogram law:

This shows the vector subtraction a - b as the addition of a and −b.

Vector Subtraction by Triangle Law



Now, let's interpret vector subtraction employing the triangle law of vector addition. We'll denote the vector drawn from the terminal point of b to the terminal point of a as c.

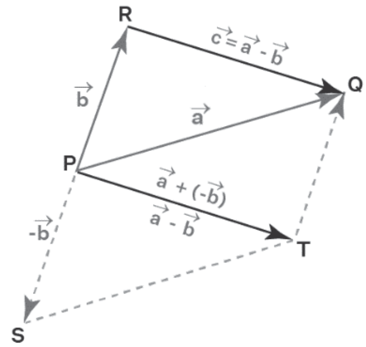
$$\vec{b} + \vec{c} = \vec{a}$$

$$\vec{c} = \vec{a} - \vec{b}$$

Note:- That b + c equals a. Therefore, c equals a - b. In simpler terms, the vector a - b is the vector drawn from the endpoint of b to the endpoint of a (assuming a and b are co-initial).
It's worth mentioning that both methods described above (utilizing the parallelogram law and the triangle law) yield the same vector for a - b. This is further illustrated in the diagram below:

Vector Subtraction by Parallelogram and Triangle Laws

The vector PT is acquired by applying the parallelogram law to add a and -b. The vector RQ is obtained by drawing the vector from the endpoint of b to the endpoint of -a. It's evident that both vectors are identical, as they possess the same magnitudes and directions.



Subtract Vectors

Here are various methods for vector subtraction:
Graphically subtracting two vectors a and b (i.e., finding a - b): Initially, ensure they are co-initial, then draw a vector from the endpoint of b to the endpoint of a.
We can execute vector subtraction a - b by adding -b (the negative of vector b, obtained by multiplying b by -1) to a. That is, a - b = a + (-b).
If the vectors are represented in component form, we simply subtract their corresponding components in the order of vector subtraction.

Properties of Vector Subtraction

Below are key characteristics of vector subtraction.
Subtracting any vector from itself yields a zero vector. In other words, a - a equals 0, for any vector a.
Vector subtraction is not commutative. This means that a - b is not necessarily equal to b - a.
Vector subtraction is not associative. In other words, (a - b) - c may not be equal to a - (b - c).

$$(a - b) \cdot (a + b) = |a|^2 - |b|^2$$
$$(a - b) \cdot (a - b) = |a - b|^2 = |a|^2 + |b|^2 - 2a \cdot b$$

Range of Resultant Problems Based on Each Topic.

Velocity

The velocity of a moving object is represented by a vector pointing in the direction of motion, with a magnitude equal to the speed.

Ex. A ball is launched with an initial velocity of 70 feet per second at an angle of 35° with the horizontal. Determine the vertical and horizontal components of the velocity.

Sol. Let v represent the velocity and use the given information to write v in unit vector form:

$$v = 70(\cos(35^\circ))i + 70(\sin(35^\circ))j$$

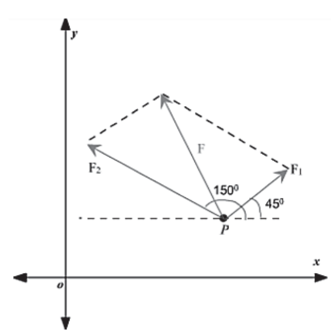
Simplify the scalars, we get:

$$v \approx 57.34i + 40.15j$$

Since the scalars are the horizontal and vertical components of v,
Therefore, the horizontal component is 57.34 feet per second and the vertical component is 40.15 feet per second.

Force

Force is similarly depicted as a vector. When multiple forces act upon an object, the resultant force felt by the object is the vectorial sum of these forces.



Ex. At point P, two forces, F_1 and F_2, with magnitudes of 20 and 30 pounds, respectively, are applied to an object as illustrated. Determine the resultant force acting at point P.

Sol. First we write F_1 and F_2 in component form:
$v \approx 57.34i + 40.15j$

Simplify the scalars, we get:

$$F_1 = (20\cos(45°))i + (20\sin(45°))j$$
$$= 20(\tfrac{\sqrt{2}}{2})i + 20(\tfrac{\sqrt{2}}{2})j$$
$$= 10\sqrt{2}i + 10\sqrt{2}j$$
$$F_2 = (30\cos(150°))i + (30\sin(150°))j$$
$$= 30(-\tfrac{\sqrt{3}}{2})i + 30(\tfrac{1}{2})j$$
$$= -15\sqrt{3}i + 15j$$

So, the resultant force F is

$$F = F_1 + F_2$$
$$= (10\sqrt{2}i + 10\sqrt{2}j) + (-15\sqrt{3}i + 15j)$$
$$= (10\sqrt{2} - 15\sqrt{3})i + (10\sqrt{2} + 15)j$$
$$\approx -12i + 29j$$

Work

The work, denoted as W, performed by a force F when moving along a vector D is given by the dot product of F and D, expressed as $W = F \cdot D$.

Ex. A force is given by the vector F = (2,3) and moves an object from the point (1,3) to the point (5,9) find the work done.

Sol. First we find the Displacement.

The displacement vector is

$$D = \langle 5-1, 9-3 \rangle = \langle 4,6 \rangle$$

By using the formula, the work done is

$$W = F \cdot D = \langle 2,3 \rangle \cdot \langle 4,6 \rangle = 26$$

If the unit of force is pounds and the distance is measured in feet then the work done is 26 ft - lb.

Ex. Find the resultant of the vector $4i + 3j - 5k$ and $8i + 6j - 10k$.

Sol. The given two vector are:

$$A = 4i + 3j - 5k \text{ and } B = 8i + 6j - 10k$$

The direction rations of the two vector are in equal proportion and hence the two vectors are in the same direction.

The following resultant vector formula can be used here.

$$R = A + B$$
$$= (4i + 3j - 5k) + (8i + 6j - 10k)$$
$$= 12i + 9j - 15k$$

Hence the resultant of the two vectors is $12i + 9j - 15k$.

Ex. Find the resultant of the vectors having magnitude of 5 units, 6 units and here inclined to each other at an angle of 60°.

Sol. The two vector are A = 5 units, B = 6 units and the angle $\emptyset = 60°$

The resultant vector can be obtained by the following formula.

$$R^2 = A^2 + B^2 + 2AB\cos\emptyset$$
$$= 5^2 + 6^2 + 2 \times 5 \times 6 \times \cos 60°$$
$$= 25 + 36 + 60 \times \tfrac{1}{2}$$
$$= 61 + 30$$
$$R^2 = 91$$
$$R = \sqrt{91}$$

Therefore the resultant vector is $\sqrt{91}$.