INTEGRATION AND ITS APPLICATIONS

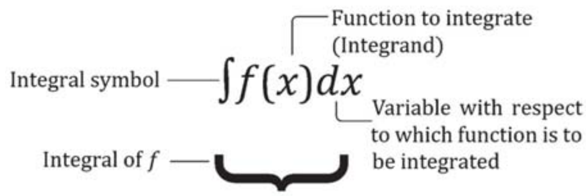
Integration

It is the reverse operation of differentiation.

We can derive the function f(x), when the derivative of the function is given through integration.

A constant ‘C’ is added to the integrand of a function. Hence,

$$\int f(x)dx = f_1(x) + C$$



Integral Values of Some Functions

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x\tan x$$

$$\frac{d}{dx}[\cot x] = -\mathrm{cosec}^2 x$$

$$\frac{d}{dx}[\mathrm{cosec}\ x] = -\mathrm{cosec}^{x\cot x}$$

$$\frac{d}{dx}[\mathrm{In}\ x] = \frac{1}{x}$$

$$\int 0dx = C$$

$$\int kdx = kx + C$$

$$\int \cos xdx = \sin x + C$$

$$\int -\sin xdx = \cos x + C$$

$$\int \sec^2 xdx = \tan x + C$$

$$\int \sec^x \tan xdx = \sec x + C$$

$$\int -\mathrm{cosec}^2 xdx = \cot x + C$$

$$\int -\mathrm{cosec}^x \cot xdx = \mathrm{cosec}\ x + C$$

$$\int \frac{1}{x}dx = \ln x + C$$

$$\int e^x dx = e^x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + Cn \neq -1$$

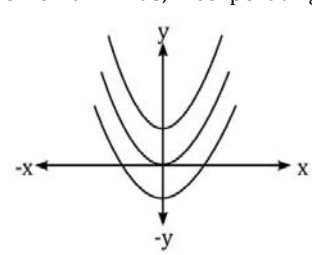
$$\int kx^n = \frac{kx^{n+1}}{n+1} + Cn \neq -1$$

$$\int x^{-1}dx = \ln x + C$$

Properties of Integration

The derivatives of any constant value within the initial function become null. Thus, incorporating 'c' into the outcome of integration yields the most comprehensive scenario.

If $\frac{dy}{dx} = 2x \Rightarrow y = x^2 + c \rightarrow$ Family of curves



Integration of polynomial function

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c; n \neq -1$$

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + c; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + c; n \neq -1$$

$$\int x^{-1} dx = \ln x + c; n \neq -1$$

When you integrate a constant multiplied by a function without limits, you get the constant multiplied by the integral of the function.

$$\int af(x)dx = a\int f(x)dx; \text{ where a is constant.}$$

When you integrate an equation with several functions added together, you can find the integral of each function separately and then add them all up.

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int [af(x) + bg(x)]dx = a\int f(x)dx + b\int g(x)dx;$$

where a and b are constant.

Geometrical Meaning of Integration

The geometrical meaning of integration is closely tied to the concept of area under a curve. Integration is essentially a mathematical operation that allows you to find the accumulation of quantities over a certain interval. In terms of geometry, integration is often used to calculate the area bounded by a curve and the x-axis within a given interval.

Here's a breakdown of the geometrical meaning of integration:

Area under a Curve: Consider a function $f(x)$ defined on an interval $[a,b]$. When you integrate $f(x)$ with respect to x over this interval, denoted by $\int abf(x)\,dx$, you're essentially finding the area of the region bounded by the curve $y=f(x)$ and the x-axis between the points $x=a$ and $x=b$.

Partitioning: To calculate this area, the interval $[a,b]$ is often divided into smaller subintervals, and the area under the curve within each subinterval is approximated. The approximation becomes more accurate as the width of the subintervals decreases, approaching zero, using methods like Riemann sums or calculus techniques like limits.

Summation of Areas: Integration sums up the areas of these approximated rectangles, trapezoids, or other shapes that collectively cover the region under the curve. As the width of the subintervals approaches zero, this summation converges to the exact area under the curve.

Negative Area: If the curve dips below the x-axis within the interval, the area is considered negative because it's below the x-axis. When calculating the integral, this is represented by negative values, indicating a decrease in the accumulated quantity.

Geometric Interpretation Beyond Area: While the area under the curve is the most common geometric interpretation of integration, integration can represent other geometric quantities as well. For instance, integrating a velocity function over a time interval gives the displacement, and integrating a density function over a volume gives the mass.

In essence, integration geometrically represents the accumulation or aggregation of quantities, often visualized as the area under a curve, but it can represent various geometric measures beyond just area.

Definite Integrals, Properties of Definite Integrals

Definite Integrals

Definite integrals represent a fundamental concept in calculus, offering a precise method for calculating the accumulated quantity of a function over a specified interval. Unlike indefinite integrals, which yield a family of functions with an arbitrary constant, definite integrals produce a single numerical value. This value corresponds to the net area bounded by the function's graph, the x-axis, and the vertical lines defining the interval.

Mathematically, the definite integral of a function $f(x)$ over the interval $[a,b]$ is denoted by $\int f_a^b(x)\,dx$. The result represents the net signed area between the curve and the x-axis within the interval $[a,b]$.

Properties of definite integrals include linearity, meaning that the integral of a sum is the sum of the integrals, as well as additivity over intervals, which states that integrating over a union of intervals is equivalent to integrating over each interval separately and summing the results. Additionally, definite integrals obey the Fundamental Theorem of Calculus, which establishes a connection between integration and differentiation.

Definite integrals find widespread applications in various fields, including physics, engineering, economics, and statistics. They are indispensable for solving problems involving accumulation, such as computing total distance traveled, total volume, or total accumulated value over time.

Properties of Definite Integrals

Properties	Description
Property 1	$\int_j^k f(x)dx = \int_j^k f(t)dt$
Property 2	$\int_j^k f(x)g(x) = -\int_j^k f(x)g(x)$, also $\int_k^j f(x)g(x) = 0$
Property 3	$\int_j^k f(x)dx = \int_j^l f(x)dx + \int_l^k f(x)$
Property 4	$\int_j^k f(x)g(x) = \int_j^k f(j+k-x)\,g(x)$
Property 5	$\int_0^k f(x)g(x) = \int_j^k f(k-x)g(x)$
Property 6	$\int_0^{2k} f(x)dx = \int_0^k f(x)dx + \int_0^k f(2k-x)dx$ …. . If f(2k - x) = f(x)
Property 7	$\int_0^2 dx = 2\int_0^x f(x)\,dx$ …. . if $f(2k-x) = f(x)$ $\int_0^2 f(x)dx = 0$. . if $f(2k-x) = f(x)$
Property 8	$\int_{-k}^k f(x)dx = 2\int_0^x f(x)dx$ … .if $(-x) = f(x)$ or it is an even function $\int_k^k f(x)dx = 0$..if $f(2k-x) = f(x)$ or it is an odd function