APPLICATIONS OF DIMENSION ANALYSIS

Physical Quantities

The quantities that can be measured by an instrument and by means of which we can describe the laws of physics are known as physical quantities. Physical quantities are used to define the material, space, time, and energy.

Process of measurement of physical quantities

Physical quantities are measured as the numerical value and supporting standard units for that numerical value. Units are necessary to define any physical quantity.

Example:

Length is a physical quantity. We can measure the height of Burj Khalifa tower as 828 meters.

Types of physical quantities

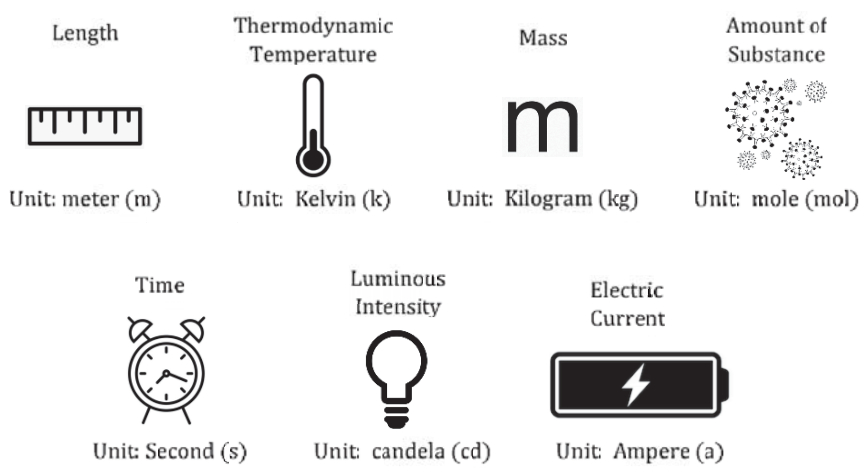
- Fundamental
- Derived
- Supplementary

Fundamental physical quantities

Certain physical quantities are used to express all the physical quantities. Such quantities are known as fundamental, absolute, or base quantities.

- They are independent of each other and cannot be obtained from one another.
- All other quantities may be expressed in terms of fundamental quantities.

Following are the seven fundamental quantities with their general units.



The amount of substance is measured in terms of the number of moles. One mole of a substance is equal to $6.02214076 \times 10^{23}$ number of particles of that substance. Here, particles can be atoms, molecules, ions, or electrons.

Derived physical quantities

Physical quantities that can be expressed as combinations of base quantities are known as derived quantities.

Example:

Speed can be written as distance per unit time. It can be expressed in unit ms –1. Similarly, velocity, acceleration, force, momentum, pressure, energy etc. can be expressed as a combination of fundamental quantities. Thus, these are all derived quantities.

Supplementary physical quantity

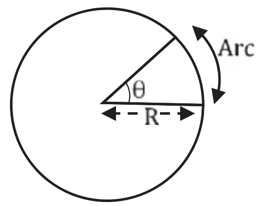
There are two quantities that have units but no dimensions. These quantities cannot be derived from fundamental quantities. These are known as supplementary quantities.

1. **Plane angle**

Angle (θ) is defined as

$$\theta = \frac{\text{Arc length}}{\text{R}}$$

Where R is the radius of the arc. Unit of angle is radian

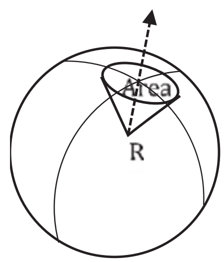


2. **Solid angle**

Solid angle (Ω) is defined as

$$\Omega = \frac{\text{Area}}{\text{R}^2}$$

Where R is the radius of the arc. Unit of solid angle is steradian



Rules for writing units of physical quantities

Symbols for units of physical quantities are printed/written in Roman (upright type), and not in italics. For example, 1 N is correct but 1 N is incorrect

Note:

This rule is followed strictly for scientific papers. In lower grades, this rule is sometimes not followed. However, for board exams, this rule should be followed.

- Unit is never written with a capital initial letter when it is written in full form, even if it is named after a scientist.
- For example, SI unit of force is newton.
- For a unit named after a scientist, the symbol or notation is a capital letter. However, for other units, the symbol is not a capital letter.

Magnitude of a quantity

- The magnitude of a physical quantity is equal to the numerical value.
  Example: Mass = 5 kg. Here, 5 is the numerical value and kg is the unit.
- The magnitude of a physical quantity is always constant, even though it can be expressed as different combinations of the numerical value and the unit. Let a quantity have numerical value of n_1 for unit u_1 and n_2 for unit u_2. Then, $n_1 u_1 = n_2 u_2$ or it can be written as

$$\text{Numerical value } \propto = \frac{1}{\text{Unit}}$$

Example:

1000 mm = 100 cm = 1 m We used some general units to define the physical quantities. In physics, there are a lot of quantities and their units can be written in different conventional systems.

Hence, the definition of units and its system should be understood properly before proceeding to other chapters.

Limitations of Dimensional Analysis

The value of the dimensionless constants cannot be calculated.

The equation containing trigonometric, exponential, and logarithmic terms cannot be analyses.

If a physical quantity depends on more than three factors, then the relation among them cannot be established because we can only get three equations by equating the powers of M, L, and T.

Dimensional Analysis

The dimensional formula can be used to

1. To check the correctness of the equation.
2. Convert the unit of the physical quantity from one system to another.
3. Deduce the relation connecting the physical quantities.

Principle of Homogeneity

According to the principle of homogeneity of dimensions, all the terms in a given physical equation must be the same.

Example: $s = ut + (\frac{1}{2})att^2$

Dimensionally $[L] = [LT^{-1} \cdot T] + [LT^{-2} \cdot T^2][L] = [L] + [L]$

Defects of Dimensional Analysis

1. While deriving the formula the proportionality constant cannot be found.
2. The equation of a physical quantity that depends on more than three independent physical quantities cannot be deduced.
3. This method cannot be used if the physical quantity depends on more parameters than the number of fundamental quantities.
4. The equations containing trigonometric functions and exponential functions cannot be derived

Points to Remember

Those quantities which can describe the laws of physics are called the physical quantity.

Example: -

- Length, mass and time.
- Physical quantities can be classified as fundamental quantities and derived quantities.
- The reference standard used to measure the physical quantities is called the unit. Units are classified as fundamental units and derived units.
- SI system is the most commonly used system of units.
- The SI is based on seven basic units and two supplementary units.
- The dimensional formula of any physical quantity is the formula that tells which of the fundamental units have been used for the measurement of that physical quantity. The dimensional formula follows the principle of homogeneity

Ex. Consider density (D), area (A), and velocity (V) as fundamental quantities, and write the dimensional formula for force (F).

Sol. We know that the dimensions of the given quantities are as follows

$$[D] = [ML^{-3}]$$
$$[V] = [LT^{-1}]$$
$$[A] = [L^2]$$
$$\text{Let, Force, } F = kD^aA^bV^c$$
$$[F] = [k][D]^a[A]^b[V]^c$$
$$[MLT^{-2}] = [ML^{-3}]^a[L^2]^b[LT^{-1}]^c$$
$$[MLT^{-2}] = [MaL^{-3a} + 2b + cT^{-c}c]$$

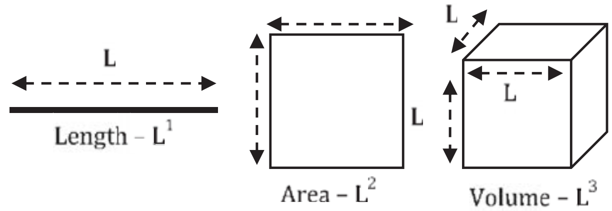
On comparing, we get

$$a = 1, c = 2 \text{ and } -3a + 2b + c = 1 \to b = 1$$

So, $F = kDAV^2$

Dimensions

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.



Symbols for dimensions of fundamental quantities

Following are the symbols used for writing dimensions of fundamental quantities which are used combined to express derived quantities as well.

Fundamental Quantities	Dimensional Symbol
Mass	M
Length	L
Time	T
Electric current	A
Temperature	K
Luminous intensity	cd
Mole	mol

Dimension Formulae of Certain Quantities

Physical Quantity	Mathematical Formula	SI Unit	Dimension Formula
Area	$l \times b$	m^2	$[L^2]$
Volume	$l \times b \times h$	m^3	$[L^3]$
Velocity	$\frac{s}{t}$	ms^{-1}	$[LT^{-1}]$
Momentum	mv	kg ms^{-1}	$[MLT^{-1}]$
Force	ma	kg ms^{-2} (N)	$[MLT^{-2}]$
Work done	Fs	kg m^2s^{-2} (J)	$[ML^2 T^{-2}]$
Displacement	L	m	[L]
Acceleration	$\frac{V}{t}$	ms^{-2}	$[LT^{-2}]$
Kinetic energy	$\frac{1}{2}mv^2$	kg m^2s^{-2} (J)	$[ML^2 T^{-2}]$
Pressure	$\frac{F}{A}$	kg $m^{-1}s^{-2}$ (Nm^{-2})	$[ML^{-1}T^{-2}]$
Torque	Fr	kg m^2s^{-2}	$[ML^2 T^{-2}]$
Moment of inertia	mR^2	kg m^2	$[ML^2]$
Angular displacement	θ	radian	Dimensionless
Angular velocity	$\frac{\theta}{t}$	rad s^{-1}	$[T^{-1}]$
Stress	$\frac{F}{A}$	kg $m^{-1}s^{-2}$	$[ML^{-1}T^{-2}]$
Strain	$\frac{\Delta l}{l}$	Unit less	Dimensionless
Charge	$I \times t$	coulomb	[AT]

Dimensional Correctness of Equation

Every physical equation should be dimensionally balanced.

To check the dimensional correctness of a given physical equation, we use the principle of homogeneity.

Dimensionally correct equation may or may not be physically correct

Principle of homogeneity

This principle states that the dimensions of all the terms in a physical expression should be the same. Example:

$$s = ut + \frac{1}{2}at^2$$
$$[L] = [LT^{-1}][T] + [LT^{-2}][T^2]$$
$$[L] = [L] + [L]$$

Ex. Show that the expression of the time period T of a simple pendulum of length l given by $T = 2\pi\sqrt{\frac{l}{g}}$ Is dimensionally correct

Sol. Dimension of LHS = [T]

Dimensional Formula

The dimensional formula of any physical quantity is the formula that tells which of the fundamental units have been used for the measurement of that physical quantity.

Dimensional formula is written for a physical quantity

1. The formula of the physical quantity must be written. The quantity must be on the left-hand side of the equation.

2. All the quantities on the right-hand side of the formula must be written in terms of fundamental quantities like mass, length and time.

3. Replace mass, length and time with M, L and T.

4. Write the powers of the terms.

Characteristics of Dimensions

1. Dimensions do not depend on the system of units.

2. Quantities with similar dimensions can be added or subtracted from each other.

3. Dimensions can be obtained from the units of the physical quantities and vice versa.

4. Two different quantities can have the same dimension.

5. When two dimensions are multiplied or divided it will form the dimension of the third quantity.

Uses of Dimension

1. It is used in conversion of units.

2. Dimensional correctness of equation: Dimension of both LHS and RHS in an equation should be the same.

3. It establishes a relation between physical quantities.

Conversion of units

Dimension is used to convert the numerical value of a physical quantity from one system of units into the other system.

Example:

Velocity from CGS system (100 cm s^{-1}) to SI system (1 ms^{-1})

The product of the numerical value (n) and its corresponding unit (u) is always constant.

$$n[u] = \text{Constant}$$
$$n_1[u_1] = n_2[u_2]$$
$$100 \text{ cm s}^{-1} = n_2 \times \text{ms}^{-1}$$
$$100 \text{ cm s}^{-1} = n_2 \times 100 \text{ cm s}^{-1}$$
$$n_2 = 1$$