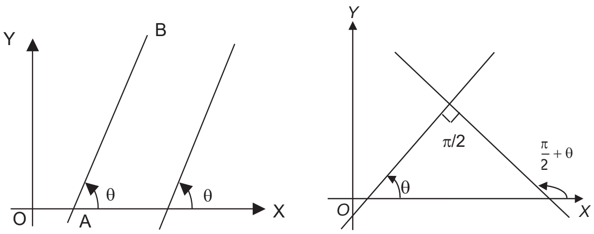
SLOPE OF A LINE

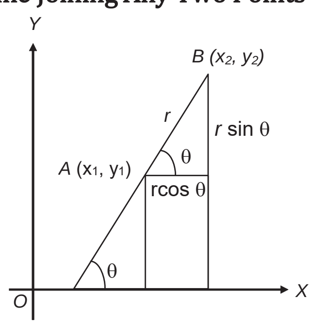


When considering a straight line AB and its angle θ with respect to the positive direction of the x-axis, the tangent of θ is defined as the slope or gradient of the straight line and is typically represented by the letter 'm.'

Consequently:

1. If two lines are parallel, they share the same slope because they both have an equal inclination to the positive x-axis.
2. If two lines are perpendicular, the product of their slopes is -1. When one line is inclined at an angle θ to the x-axis, the other must be inclined at (90° + θ), making their slopes tan θ and tan (90° + θ), which simplifies to tan θ and -cot θ. Therefore, their product equals -1.

1. To Find The Slope Of The Line Joining Any Two Points (X_1, Y_1) And (X_2, Y_2)



Consider the line segment connecting the points, which has a length of r, and let the line be oriented at an angle θ relative to the x-axis.

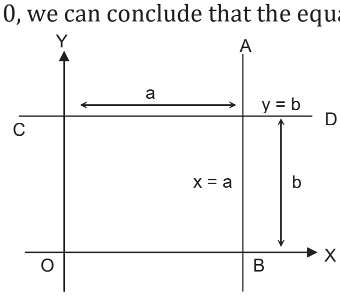
$$r\cos\theta = x_2 - x_1$$
$$r\sin\theta = y_2 - y_1$$
$$\tan\theta = \frac{y_2-y_1}{x_2-x_1}$$

Thus, the desired slope is given by tan θ. Hence, the formula for the slope is as follows:

$$m = \frac{y_1-y_2}{x_1-x_2} = \frac{\text{difference of the ordinates of the two points}}{\text{difference of the abscissa}}$$

2. Lines Parallel To The Co Ordinate Axes

Consider line AB, which runs parallel to the Y-axis and is positioned at a distance 'a' from it. At every point along AB, the x-coordinate is consistently 'a,' thus leading to the equation x = a for AB. When we substitute a = 0, we can conclude that the equation for the y-axis is x = 0.



Likewise, the equation for the straight line CD, which runs parallel to the X-axis and is located at a distance 'b' from it, is $y = b$. When we set $b = 0$, we can infer that the equation for the X-axis is $y = 0$.

3. Angle Between Two Given Straight Lines

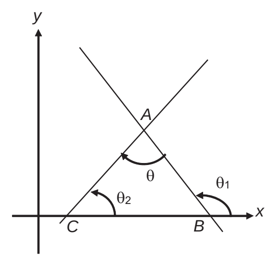
Consider lines AB and AC, with slopes m_1 and m_2 respectively, and they are oriented at angles θ_1 and θ_2 relative to the X-axis.

$$\angle = \theta_1 - \theta_2 = \theta$$

$$\tan \angle CAB = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

Here, θ represents the angle between AB and AC.



It is common practice to define θ as the acute angle between the two lines. Therefore, in most cases, one can use the following formula:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

When the lines are parallel,

$$\tan\theta = 0, \text{ since } \theta = 0$$

$$m_1 - m_2 = 0$$

$$m_1 = m_2$$

If the lines are perpendicular, $\tan\theta$ is undefined because $\theta = \frac{\pi}{2}$

$$m_1 m_2 = +1 = 0$$

$$m_1 m_2 = -1$$

Ex. Determine the acute angle between the two lines with the slopes $\frac{1}{5}$ and $\frac{3}{2}$

Sol. When the angle between the lines is denoted as θ,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{\frac{1}{5} - \frac{3}{2}}{1 + (\frac{1}{5}) \times (\frac{3}{2})}\right| = |-1| = 1$$

Hence, the angle measures 45 degrees.