

POSITION OF TWO POINTS RELATIVE TO A LINE

Let us consider two points $A = (X_1, Y_1)$, $B = (X_2, Y_2)$, and the line $ax + by + c = 0$ and $S_1 = ax + by + c$, $S_2 = ax_2 + by_2 + C$.

1. If $S_1 = 0$ and $S_2 = 0$, then both points lie on the line.
2. If $S_1 S_2 > 0$, then both points lie on the same side of the line.
3. If $S_1 S_2 < 0$, then points lie on the opposite side of the line.

Ex. Find the value of a if (a, a) and $(-a, -a)$ lies in same side of the line, $x + y + 2 = 0$.

Sol. For same side

$$\begin{aligned}
 (a + a + 2)(-a - a + 2) &> 0 \\
 (a + 1)(-a + 1) &> 0 \\
 (a + 1)(a - 1) &< 0 \\
 a &\in (-1, 1)
 \end{aligned}$$
