EQUATION OF THE ANGLE BISECTORS

The equations of the bisectors of the angles between the straight lines.
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by,

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

1. Procedure for determining the equation of the bisector of the angle that includes the origin involves rephrasing the equations of the two lines to ensure positive constant terms. For the bisector of the angle, whether it includes or excludes the origin, positive and negative signs are considered accordingly.

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

2. Working rule for finding acute (internal) and obtuse (external) angle bisectors
We make the constant terms in the equations of the given straight lines
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ Positive if they are negative by suitable multiplication i.e., $c_1 > 0$ and $c_2 > 0$.

- Then field $a_1a_2 + b_1b_2$. If $a_1a_2 + b_1b_2 > 0$ then positive sign of

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \dots \text{ (i)}$$

Will give the obtuse angle bisector and the negative sign, the acute angle bisector.

- If $a_1a_2 + b_1b_2 < 0$ the negative sign of (i) will give obtuse angle bisector and the positive sign of (i) will give acute angle bisector.
- If $a_1a_2 + b_1b_2 = 0$ then both bisector will be right angle bisector.

3. Working rule to find the equations of the internal angle bisectors of a triangle
We first find the coordinates of the vertices by solving the equations of the sides two by two. Then we substitute the coordinates of each vertex in the equation of the opposite side, if it is positive, we multiply the equation by 1, but if the result is negative, multiply the equation of the side by -1.
Finally we write the equation of the bisectors of angles between any two of the sides taking positive sign before the radical sign.

Ex. For the straight line $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the

1. Bisector of the angle which contains the origin.
2. Bisector of the obtuse angle between them.
3. Bisector of the acute angle between them.

Sol. **1.** For point (0, 0), $4x + 3y - 6$ and $5x + 12y + 9$ are of opposite signs.
Hence equation of the bisector of the angle between the given straight lines containing the origin is given by.

$$\frac{4x+3y-6}{5} = -\frac{5x+12y+9}{13}$$
$$52x + 39y - 78 = -25x - 60y - 45$$
$$7x + 9y - 3 = 0$$

2. Writing the equation of the straight lines so that constants become positive. We have

$$-4x - 3y + 6 = 0$$
$$5x + 12y + 9 = 0$$

The equations of the bisectors of the angle between given straight lines are given by

$$\frac{5x+12y+9}{13} = \pm\frac{-4x-3y+6}{5} \quad \dots (1)$$

Here we observe that,

$$aa^2 + bb = -20 - 36 = -56 < 0$$

Hence taking negative sign of (i) we get the equation of obtuse angle bisector as.

$$25x + 60y + 45 = 52x + 39y - 78$$
$$27x - 21y - 123 = 0$$
$$9x - 7y - 41 = 0$$

3. Taking positive sign of (i) we get, the required equation of the acute angle bisector as

$$7x + 9y - 3 = 0$$