EQUATIONS IN DIFFERENT FORMS OF A STRAIGHT LINE

As mentioned earlier, in coordinate geometry, every curve is expressed through an equation that establishes a relationship between x and y, the coordinates of any point on that curve. Similarly, a straight line also has an equation to represent it. It becomes evident that to represent a straight line accurately, we require two distinct pieces of information. The equation changes depending on the specific information provided.

Hence, we have various forms of equations for straight lines as follows:

Slope One Point Form

If a line possesses a slope ($m = \tan\theta$), it indicates the direction of the line. It should be noted that 'm' by itself is insufficient to determine the equation of the line. With the same slope, multiple straight lines can exist, all of which are parallel.

However, if it is known that the line passes through a specific point (x_1, y_1), then the equation takes the following form:

$$y_2 - y_1 = m(x - x_1)$$

Ex. If a line has a slope of $\frac{1}{2}$ and goes through the point (-1, 2), determine its equation.

Sol. The equation representing the line

$$y - 2 = \frac{1}{2}(x - (-1))$$

$$2y - 4 = x + 1$$

$$x - 2y + 5 = 0$$

Y-Intercept Form

If the slope is denoted as 'm' and the length 'c' (referred to as the intercept) on the y-axis is provided, then the equation takes the following form:

$$y = mx + c$$

Ex. Determine the equation of a line that has a slope of $\frac{1}{2}$ and intersects the positive y-axis at a length of $\frac{5}{2}$.

Sol.

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$2y = x + 5$$

$$x - 2y + 5 = 0$$

Two Point Form

The equation for a line that passes through two points, denoted as (x_1, y_1) and (x_2, y_2), takes the following form: $\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$

In this equation, x_1 is not equal to x_2, and $(\frac{y_1-y_2}{x_1-x_2})$is the slope (m) of the line.

Ex. Determine the equation of a line that goes through the points (1, 5) and (3, 7).

Sol.

$$\frac{y-5}{x-1} = \frac{5-7}{1-3} = \frac{-2}{-2} = 1$$

$$y - 5 = x - 1$$

$$x - y + 4 = 0$$

Symmetric Form

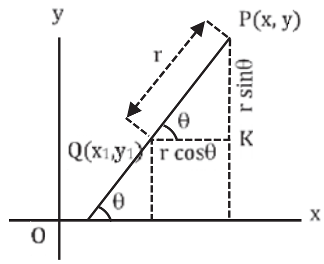
If (x_1, y_1) represents a specified point on a straight line, (x, y) denotes any point on that line, θ signifies the inclination of the line, and 'r' indicates the distance between the points (x, y) and (x_1, y_1), then the following relationship holds:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Consider point P represented by (x, y) as any point on the line. Now, let's take point Q with coordinates (x_1, y_1), such that the distance PQ is equal to 'r.' We'll draw lines from both Q and P that are parallel to the QY direction, and we'll also draw a line QK parallel to the OX axis.

From the triangle QKP, we can derive the following relationship:

$$QK = PQ\cos\theta$$



$$x - x_1 = r\cos\theta$$
$$\frac{x-x_1}{\cos\theta} = r$$
$$PK = PQ\sin\theta$$
$$y - y_1 = r\sin\theta$$
$$\frac{y-y_1}{\sin\theta} = r$$
$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

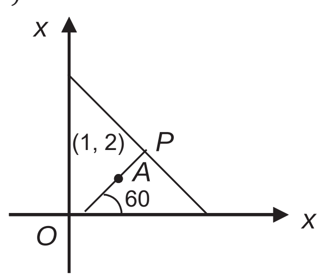
This is known as the symmetric form of the line equation. It proves to be particularly valuable when dealing with the length (r) of a segment of the line.

Ex. A straight line passes through a point A (1, 2) and forms a 60-degree angle with the x-axis. This line intersects the line $x + y = 6$ at point P. Determine the length of AP.

Sol. Consider the line we are looking for to be represented by the equation:

$$\frac{x-1}{\cos 60^\circ} = \frac{y-2}{\sin 6^\circ} = r(\equiv AP)$$

P is $(1 + r\cos 60^\circ, 2 + r\sin 60^\circ)$

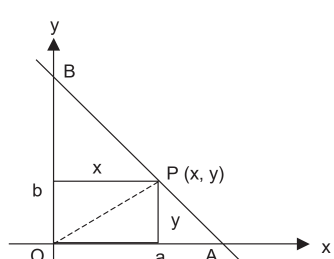


This line also lies on the equation $x + y = 6$, and the condition is:

$$\left(1 + \frac{r}{2}\right) + \left(2 + r\frac{\sqrt{3}}{2}\right) = 6$$
$$r\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 3$$
$$r = \frac{6}{1+\sqrt{3}} = \frac{6(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$
$$r = \frac{6(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{6(1-\sqrt{3})}{-2}$$
$$r = 3(\sqrt{3} - 1).$$

Intercept Form

Suppose the line intersects the x and y-axes, resulting in intercepts 'a' and 'b,' respectively. Let P(x, y) represent any arbitrary point on this line. Now, we'll connect point O to point P, creating segment OP.



In this context, we can state:
Area of triangle OAB = area of triangle OAP + area of triangle OPB

$$\frac{1}{2}ab = \frac{1}{2}ay + \frac{1}{2}bx$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is referred to as the intercept form of a straight line equation.

Ex. Determine the equation of the straight line that goes through the point (3, 4) and has an intercept on the y-axis twice the size of its intercept on the x-axis.

Sol. Consider the equation of the line to be:

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{... (1)}$$

As per the question, where $b = 2a$, it follows that, based on equation (i), the equation of the line will be:

$$\frac{x}{a} + \frac{y}{2a} = 1$$

$$2x + y = 2a \quad \text{.... (2)}$$

Since line (2) passes through the point (3, 4)

$$2 \times 3 + 4 = 2a$$

$$a = 5$$

From (2), equation of required line will be

$$2x + y = 10$$

Normal Form

If 'p' represents the length of the perpendicular line from the origin to a straight line, and 'α' is the angle at which the perpendicular intersects the x-axis, the equation of the straight line can be derived as follows:

Consider the straight line as PQ, with OQ equal to 'p,' representing the perpendicular line drawn from the origin O. Let ∠QOX be denoted as 'α.'

Now, draw the ordinate PN, and also draw NR as a perpendicular line to OQ, and PM as a perpendicular line to RN.

This leads us to the following relationship:

$$\angle PNM = 90^\circ - \angle RNO = \alpha$$

$$P = 0Q = 0R + RQ = 0O + PM$$

$$= 0N\cos\alpha + PN\sin\alpha$$

$$P = x\cos\alpha + y\sin\alpha$$

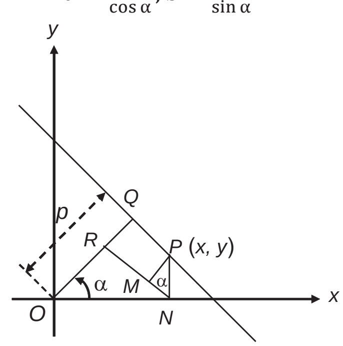
$x\cos\alpha + y\sin\alpha = p$ is the required equation.

This is called the perpendicular form.

Alternatively,

Assume that $\frac{x}{a} + \frac{y}{b} = 1$ represents the equation of the straight line. In

In this case, $a = \frac{p}{\cos\alpha}, b = \frac{p}{\sin\alpha}$



By substitution, the equation becomes

$$\frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$$

$$x\cos\alpha + y\sin\alpha = p$$

This is known as the normal form of the equation to a straight line.

Ex. Determine the equation of the straight line for which the length of the perpendicular from the origin measures $3\sqrt{2}$ units, and this perpendicular creates a 75-degree angle with the positive direction of the x-axis.

Sol. Consider line AB as the desired line, with OL serving as the perpendicular.

Given $OL = 3\sqrt{2}$ and $\angle LOA = 75^\circ$

Equation of line AB will be

[Normal form] $x\cos 75^\circ + y\sin 75^\circ = 3\sqrt{2}$ ... (1)

$$\cos 75^\circ = \cos(30^\circ + 45^\circ)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\sin 75^\circ = \sin(30^\circ + 45^\circ)$$

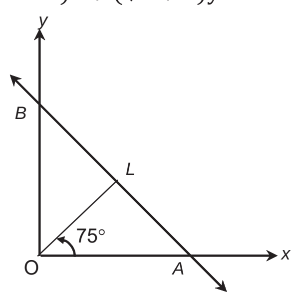
$$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

From (i) equation of line AB is

$$x\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + y\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = 3\sqrt{2}$$

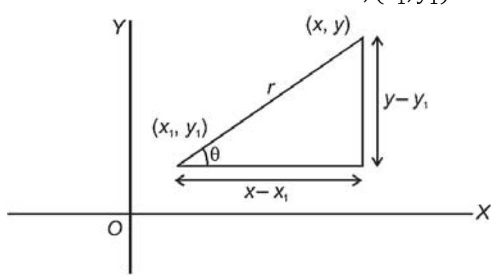
$$(\sqrt{3}-1)x + (\sqrt{3}+1)y - 12 = 0$$



Parametric Form

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$$

Where (x, y) is a point on the line at a distance of 'r' from, (x_1, y_1) and $\tan\theta$ is the slope of the line.



Here $x - x_1 = r\cos\theta$ and $y - y_1 = r\sin\theta$

$$\Rightarrow \frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$$

As three may be two position of point (x, y) with respect to (x_1, y_1).

General point on the line is $x = x_1 \pm r\cos\theta$ and $y = y_1 \pm r\sin\theta$.

Ex. Find the coordinate of the point on the line $-x + 1 = 0$ which is it a distance of 3 units from (1,0)

Sol. $x_1 = 1, y_1 = 0$

$$\tan\theta = 1, \Rightarrow \theta = \frac{\pi}{4}$$

$$r = 3$$

$$x = x_1 \pm r\cos\theta = 1 \pm 3 \times \cos 45^\circ = 1 \pm \frac{3}{\sqrt{2}}$$

$$y = y_1 \pm r\sin\theta = 0 \pm 3 \times \sin 45^\circ = \pm\frac{3}{\sqrt{2}}$$

Hence the point is $(1 + \frac{3}{\sqrt{2}}, +\frac{3}{\sqrt{2}}), (1 - \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$

General Form

Any linear equation of the first degree in both x and y, expressed as $Ax + By + C = 0$, depicts a straight line.

The straight line in this form, has

(a) Slope $= m = -\frac{A}{B} = -\frac{\text{coefficient of x}}{\text{coefficient of y}}$

(b) x Intercept $= -\frac{C}{A}$

And y intercept $= -\frac{C}{B}$

Ex. Determine the value of 'k' for which the straight lines represented by the equations $2x + 3y + 4 + k(6x - y + 12) = 0$ and $7x + 5y - 4 = 0$ are perpendicular to each other.

Sol. Given lines are

$$(2 + 6k)x + (3 - k)y + 4 + 12k = 0 \quad \ldots (1)$$

$$7x + 5y - 4 = 0 \quad \ldots (2)$$

Slope of line (1), $m_1 = -\frac{2+6k}{3-k} = \frac{2+6k}{k-3}$

And slope of line (2), $m_2 = -\frac{7}{5}$

Because line (1) is perpendicular to line (ii),

$$(\frac{2+6k}{k-3})(-\frac{7}{5}) = -1$$

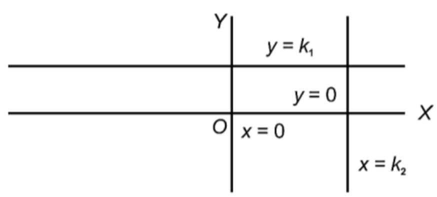
$$(2 + 6k)7 = 5(k - 3)$$

$$14 + 42k = 5k - 15$$

$$37k = -29$$

$$k = -\frac{29}{37}$$

Equation of Lines Parallel To Co-ordinate Axes



1. The equation of the x-axis is given by $y = 0$.
2. The equation of the y-axis is represented as $x = 0$.
3. The equation of a line parallel to the x-axis can be expressed as $y =$ constant, denoted as k_1.
4. The equation of a line parallel to the y-axis is given by $x =$ constant, denoted as k_2.

Ex. Find the parametric form of $y = \sqrt{3}x + 4$.

Sol. Here, $\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$$x = 0, y = 4$$

Here general form $x = 0 \pm r\cos\theta = \pm r\cos\frac{\pi}{3}$

$$y = 4 \pm r\sin\frac{\pi}{3}$$