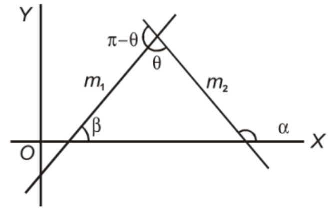
# ANGLE BETWEEN THE LINES

The slopes of the lines are m and m. The angle between the lines are $\theta$ and $\pi - \theta$




Let $m_2 = \tan\alpha, m_1 = \tan\beta$

But $\alpha$ is external to $\beta$ and $\theta \Rightarrow \alpha = \beta + \theta$

$$\theta = \alpha - \beta$$

$$\tan\theta = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta} = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan(\pi - \theta) = -\frac{m_1 - m_2}{1 + m_1 m_2}$$

Hence the angle between the lines are given by

$$\tan\phi = \pm\frac{m_1 - m_2}{1 + m_1 m_2} \text{ where } \phi = \theta, \pi - \theta$$

## Special Cases

(a) If $m_1 m_2 = -1, m_1, m_2 \in R$ then lines are perpendicular i.e. $\phi = 90°$. Also if $m_1 \to \infty$ and $m_2 = 0$ versa, the lines are perpendicular.

(b) $\phi = 0°$, then lines will be parallel and in this case $m_1 = m_2$, for real slopes.

**Ex.** Find the slope of a line whose inclination to the positive direction of x-axis in anticlockwise sense is

(a) 30° (b) 150°

**Sol.** (a) Slope $= \tan 30° = \frac{1}{\sqrt{3}}$ (b) Slope $= \tan 150° = -\cot 60° = -\frac{1}{\sqrt{3}}$

**Ex.** If a line passes through the points (1, 1) and (2, 2), calculate the angle that the line forms with the x-axis. Let the slope of the line be represented as tanθ, then...

**Sol.** $\tan\theta = \frac{2-1}{2-1} = 1 \Rightarrow \theta = 45°$

Hence, with x-axis line makes 45° and 135°

**Ex.** Determine the value of y for which the line passing through (3, y) and (2, 7) is parallel to the line passing through (-1, 4) and (0, 6).

**Sol.** Let A (3, y), B (2, 7), C (-1, 4) and D (0, 6) be the given points.

$$m_1 = \text{ slope of the line AB} = \frac{7-y}{2-3} = y - 7$$

$$m_2 = \text{ slope of line CD} = \frac{6-4}{0-(-1)} = 2$$

Since AB and CD are parallel.

$$m_1 = m_2$$

$$y - 7 = 2$$

$$y = 9$$

**Ex.** Demonstrate, without employing the Pythagorean theorem, that the points A (4, 3), B (6, 4), and C (5, 6) form the vertices of a right-angled triangle.

**Sol.** In $\triangle$ ABC,

$$m_1 = \text{ Slope of AB} = \frac{4-3}{6-4} = \frac{1}{2}$$

$$m_2 = \text{ Slope of BC} = \frac{6-4}{5-6} = -2$$

Clearly, $m_1 m_2 = -1$

This implies AB ⊥ BC, i.e., ∠ABC = 90°.

Hence proved.