ANALYSIS OF TWO LINES

1. Point of intersection

$$a_1x + b_1y + c_1 = 0 \quad \ldots (1)$$
$$a_2x + b_2y + c_2 = 0 \quad \ldots (2)$$

Point of intersection is find out by elimination method or by substitution method.

For above lines point of intersection is $\left(\frac{b_2c_1-c_2b_1}{b_1a_2-b_2a_1}, \frac{a_1c_2-a_2c_1}{a_2b_1-a_1b_2}\right)$.

2. Parallel lines

The equation of line parallel to $ax + by + c = 0$ is $ax + by + \lambda = 0, \lambda \in R$, because slope will be same

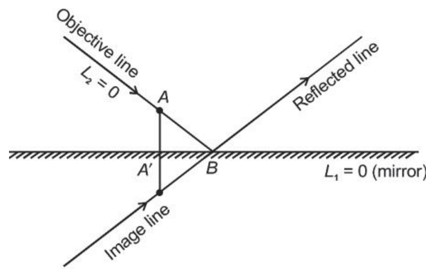
3. Perpendicular lines

The equation of the line perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda = 0, \lambda \in R$..

4. Distance between two parallel lines

The distance between $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$.

5. Image of one line through the other line mirror



To find the image of $L_2 = 0$ through $L_1 = 0$ first we find the point of intersection of $L_1 = 0$ and $L_2 = 0$. Let this is B. Again we take a point A on the line L_2, and find its image through $L_1 = 0$, let this is A'. The required line will be the line BA'.

The equation of reflected line and equation of image line are same.

Ex. Find the distance between the parallel lines 4x + 3y 11 = 0 and 8x + 6y - 15 = 0.

Sol. $4x + 3y - 11 = 0$

$$8x + 6y - 15 = 0$$
$$4x + 3y - \frac{15}{2} = 0$$

The required distance $= \frac{\left|\frac{-15}{2}-(-11)\right|}{\sqrt{4^2+3^2}}$

$$= \frac{\left|\frac{-15}{2}+11\right|}{5}$$
$$= \frac{7}{2\times5} = \frac{7}{10} \text{ units}$$

6. Family of lines

Family of lines passes through the point of intersection of $L_1 = 0$ and $L_2 = 0$ is $L + L_2 = 0, \lambda \in R$. The point of intersection of $L_1 = 0$ and $L_2 = 0$ is called fixed point for the family and is to be find out by any 1 given condition.

Ex.
1. If a, b, c are in A.P. then show that $ax + by + c = 0$ passes through a fixed point. Find the Fixed point.
2. If $9a^2 + 16b^2 - 24ab - 25c^2 = 0$, then the family of straight lines $ax + by + c = 0$ is concurrent at the points whose co-ordinates are given by
3. If 3a + 4b 5c = 0, then the family of straight lines $ax + by + c = 0$ passes through a fixed point. Find the coordinates of the point.

Sol.
1. a, b, c are in A.P
$a - 2b + c = 0$

Comparing the given equation $ax + by + c = 0$ with $a - 2b + c = 0$ we find that the straight line $ax + by + c = 0$ passes through the fixed point (1, 2).

2. We have

$$9a^2 + 16b^2 - 24ab - 25c^2 = 0$$
$$(3a - 4b)^2 - (5c)^2 = 0$$
$$(3a - 4b + 5c)(3a - 4b - 5c) = 0$$
$$\left(\frac{3}{5}a - \frac{4}{5}b + c\right)\left(\frac{-3}{5}a + \frac{4}{5}b + c\right) = 0$$

The family of lines $ax + by + c = 0$ is either concurrent at $\left(\frac{3}{5}, \frac{-4}{5}\right)$ or $\left(-\frac{3}{5}, \frac{4}{5}\right)$.

3. We have

$$3a + 4b - 5c = 0$$
$$-\frac{3}{5}a - \frac{4}{5}b + c = 0$$

The given family of lines $ax + by + c = 0$ passes through the fixed point $\left(\frac{-3}{5}, \frac{-4}{5}\right)$

Ex. Find the equations of the straight lines which pass through the point $P(x_1, y_1)$ and are inclined at an angle $\tan^{-1}(m)$ to the straight line $y = mx + c$.

Sol. The equations of the straight lines passing through (x_1, y_1) inclined at angle of $\tan^{-1}(m)$ to the straight line $y = mx + c$ are

$$y - y_1 = \frac{m - m}{1 + m \cdot m}(x - x_1)$$
$$y - y_1 = 0$$
$$y - y_1 = \frac{m + m}{1 - mm}(x - x_1)$$
$$(1 - m^2)(y - y_1) = 2m(x - x_1)$$