Chapter 8

Permutations & Combinations

INTRODUCTION

Counting is the most fundamental application of mathematics. Numerous natural methods are employed for counting. This chapter delves into various established techniques that are significantly faster than conventional counting methods. Our primary focus is on methods for counting the number of arrangements (permutations) and the number of selections (combinations), although these techniques can be applied to counting in various other situations as well.

FUNDAMENTAL PRINCIPLE OF COUNTIN (COUNTING WITHOUT ACTUAL COUNTING)

If an event A can occur in 'm' different ways and another event B can occur in 'n' different ways, then the total number of different ways for:

(a) The simultaneous occurrence of both events in a definite order is $m \times n$. This principle can be extended to any number of events, known as the multiplication principle.

(b) The happening of exactly one of the events is $m + n$, known as the addition principle.

Ex. In India, there are 15 IITs, and assuming each IIT has 10 branches, the IITJEE topper can choose both the IIT and branch in $15 \times 10 = 150$ ways.

Ex. In India, with 15 IITs and 20 NITs, a student who has cleared both IITJEE and AIEEE exams can choose an institute in a total of $(15 + 20) = 35$ ways.

Ex. There are 8 buses operating from Udaipur to Jaipur and 10 buses from Jaipur to Delhi. In how many ways can a person travel from Udaipur to Delhi via Jaipur by bus?

- Identical things of another type, r identical things of the third type, and n different things
- Number of divisors of N

- Divisions and distributions
  - Distinct objectives
  - Distribution of identical objectives
- Application of Multinomial Expansion
  - Different cases of multinomial theorem
- Exponent Of a Prime Number n!.
- Principles of inclusion and exclusion
  - Derangement
  - Distribution of n different objects into r distinct boxes if in each box at least 1object is placed

Sol. Let E_1 represent the event of traveling from Udaipur to Jaipur, and E_2 represent the event of traveling from Jaipur to Delhi by the person.
E_1 can occur in 8 ways, and E_2 can occur in 10 ways.
Since both events E_1 and E_2 are to happen in order simultaneously, the number of ways is given by $8 \times 10 = 80$.

Ex. A college provides 6 courses in the morning and 4 in the evening. Determine the number of ways a student can select exactly one course, either in the morning or in the evening.

Sol. The student has 6 options among the morning courses, allowing him to choose one course in 6 ways. Additionally, for the evening course, he has 4 choices, and he can select one in 4 ways. Therefore, the total number of ways is $6 + 4 = 10$.

PERMUTATION & COMBINATION

Factorial:

A handy Notation: $n! = n.(n-1).(n-2)..............3.\ 2.\ 1$; $n! = n.(n-1)!$ Where $n \in N$

1. $0! = 1! = 1$
2. Factorials are not defined for negative integers.
3. n! is also denoted by
4. $(2n)! = 2^n . n!\ [1.3.5.7........(2n-1)]$
5. Prime Factorization of (n): If (p) is a prime number and (n) is a positive integer, then The exponent of (p) in (n) is denoted by ($E_p(n)$) and is given by

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots .. + \left[\frac{n}{p^k}\right]$$

Where $p^k \leq n < p^{K+1}$ and [x] denotes the integral part of x.
If we express the powers of each prime contained in any number (n) individually, then (n) can be represented as $n = 2^{\alpha_i} \cdot 3^{\alpha_2} \cdot 5^{\alpha_3} \cdot 7^{\alpha_4}$
Where a_i are whole numbers.