EXPONENT OF A PRIME NUMBER N!.

1. **Highest power of a prime p in ! is denoted by**

$H_p(n!)$ or $p^i(n)$ is given by

$$H_p(n!) = \sum_{k=1}^{m} \left[\frac{n}{p^k}\right]$$

where $p^m \leq n < p^{m+1}$, where [ ] represents greatest integer function

Ex. Find the highest of 5 in (100)!

Sol. We have $$H_5(100!) = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] + \left[\frac{100}{5^3}\right]$$
$$= 20 + 4 + 0 = 24$$

2. **Highest power of a composite number m in n!.**

Let p be the largest prime factor of m. Then every prime factor q of m such that q< p must occur as many times as p occurs in n!. Hence we conclude that

$H_m(n!) = H_p(n!)$, where p is the largest prime factor of m.

Ex. Find n such that n! Ends in 12 zeros.

Sol. Let n! Ends with e zeros

Then $n \geq 4e$

Now, $e = 12 \Rightarrow n \geq 48$

They are 50, 55 etc.

Let us examine 50

We find that $H_5(50)! = \left[\frac{50}{5}\right] + \left[\frac{50}{5^2}\right] + \left[\frac{50}{5^3}\right] = 10 + 2 + 0 = 12$

Which shows that 50! Ends in 12 zeros.

Also we note that none of 51, 52, 53, 54 is divisible by 5 so none of these contribute additional zero to n!. Consequently each 501, 511, 52, 53, 54! Ends in 12 zeros.

1. From the argument above it follows that if n! ends in m zeros, then $n > 4m$. After considering n's which are multiple of the prime p and are greater than 4m, we can obtain the n's f or which n! Ends in m zeros.
2. It may be found that for a given m it is not necessary that an n must exist for which n! Ends in m zero.

   Let us verify it. Let us find n such that n! Ends in 23 zeros.

   From the above discussion $n \geq 4 \times 23 = 92$.

   The first multiple of 5 after 92 is 95 and the next multiple of 5 is 100.

   We find that

$$H_5(95!) = \left[\frac{95}{5}\right] + \left[\frac{95}{25}\right] + \left[\frac{95}{125}\right] = 19 + 3 + 0$$
$$H_5(100!) = \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] = 20 + 4 + 0$$

   Which show that 95!, 96, 97, 98, 99! Ends in 22 zeros but 100! Ends in 24 zeros. Hence there does not exist any n for which n! Ends in 23 zeros.

3. **(i) Highest power of a prime number p in $^n\mathbf{C}_r$ since $^n\mathbf{C}_r = \frac{\mathbf{n!}}{\mathbf{r!(n-r)!}}$**

We find $H_p(n!) = \alpha, H_p(r!) = \beta$ and $H_p[(n-r)!] = \gamma$.

Then $H_p(\,^nC_r) = \alpha - (\beta + \gamma)$

(ii) Highest power of a prime number p in $^n\mathbf{P}_r$

$$^nP_r = \frac{n!}{(n-r)!} = r! \cdot \,^nC_r$$

$$H_p(\,^nP_r) = H_p(n!) - H_p[(n-r)!]$$

Ex. Find the highest power of 3 in $^{50}C_{10}$& $^{50}P_{10}$

Sol. We have $^{50}C_{10} = \frac{50!}{10!40!}$

$$H_3(50!) = [\frac{50}{3}] + [\frac{50}{9}] + [\frac{50}{27}] + [\frac{50}{81}]$$
$$= 16 + 5 + 1 + 0 = 22$$
$$H_3(40!) = [\frac{40}{3}] + [\frac{40}{9}] + [\frac{40}{27}] + [\frac{40}{81}]$$
$$= 13 + 4 + 1 + 0 = 18$$
$$H_3(10!) = [\frac{10}{3}] + [\frac{10}{9}] + [\frac{10}{27}]$$
$$= 3 + 1 + 0 = 4$$
$$H_3(\,^{50}C_{10}) = H_3(50!) - H_3(40!) - H_3(10!)$$
$$= 22 - 18 - 4 = 0$$
$$H_3(\,^{50}P_{10}) = H_3(50!) - H_3(40!)$$
$$= 22 - 18 = 4$$