COMBINATION

If ^nC_r denotes the number of combinations (selections) of n different things taken r at a time,

Then, $n_{C_1} = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!}$

Where $r \leq n; n \in N$ and $r \in W$.

A. Given (n) different objects, the number of ways to select at least one of them is,

$$^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots {}^nC_n = 2^{n-1}$$

This can also be expressed as the total number of combinations of \(n \) distinct items.

B. **1.** The total number of ways to make a selection by choosing some or all from (p + q + r + ldots) items, where (p) are identical items of one kind, (q) are identical items of a second kind, (r) are identical items of a third kind, and so on, is given by:

$$(p + 1)(q + 1)(r + 1)\ldots\ldots\ldots - 1.$$

2. The total number of ways to select one or more items from (p) identical items of one kind,(q) identical items of a second kind, (r) identical items of a third kind, and (n) different items is given by:

$$(p + 1)(q + 1)(r + 1)\,2^{n-1}$$

1.) $^nC_r = {}^nC_{n-r\ r}$ 2.) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ 3.) $^nC_r = 0$ if r {0, 1, 2, 3........, n}

Ex. Fifteen players are chosen for a cricket match.

1. How many ways can the playing 11 be selected?

2. How many ways can the playing 11 be selected, including a specific player?

3. How many ways can the playing 11 be selected, excluding two particular players?

Sol. **1.** Selecting 11 players from a pool of 15

Number of ways = $^{15}C_{11} = 1365$.

2. Selecting 10 players from the remaining 14, considering one player is already included.

Number of ways = $^{14}C_{10} = 1001$.

3. Selecting 11 players from the remaining 13, excluding two particular players.

Number of ways = $^{13}C_{11} = 78$.

Ex. There are 3 books of Mathematics, 4 of Science and 5 of English. How many different collections can be made such that each collection consists of-?

1. One book of each subject?

2. At least one book of each subject?

3. At least one book of English?

Sol. **1.** $^3C_1 \times {}^4C_1 \times {}^5C_1 = 60$

2. $(2^3 - 1)(2^4 - 1)(2^5 - 1) = 7 \times 15 \times 31 = 3255$

3. $(2^5 - 1)(2^3)(2^4) = 31 \times 128 = 3968$

Properties of ^nC_r

1. Number of lines by n points in which no three points are collinear is " $^nC_2$

2. Number of lines by n points out of which m points are collinear and except these m points no three points are collinear is $^nC_2 + {}^mC_2 = 1$

3. Number of diagonals of a polygon of n side = $^nC_2 - n = \frac{n(n-3)}{2}$.

4. Number of triangle by n points in which no three are collinear = $^nC_3$

5. Number of triangle by n points out of which only m are collinear = $^nC_3 - {}^mC_3$

6. If a polygon has n sides and triangles are formed by joining the vertices then

(a) Number of triangles having one side common with the sides of polygon = $n(n - 4)$

(b) Number of triangle having two sides common with the sides of polygon is n.

(c) Number of triangle having no side common with the sides of polygon is $^nC_3 - n - n(n-4)$

7. Number of points of intersections of n lines if no three lines are concurrent and no two lines are parallel is ${}^{n}C_2$

8. Number of points of intersection of n lines if m lines are concurrent and except these m no any three are concurrent = ${}^{n}C_2 - {}^{m}C_2 + 1$

9. Maximum number of points of intersection of n circles= 2. ${}^{n}C_2$.

10. Maximum number of point of intersection of n lines and m circuit =

$$ {}^{n}C_2 + 2 \cdot {}^{m}C_2 + 2 \cdot {}^{n}C_1 \cdot {}^{m}C_1. $$

11. Number of parallelogram by n parallel lines of one set and m parallel lines of another set is ${}^{n}C_2\,{}^{m}C_2$

12. Number of rectangle of any size in a square of size

$$ n \times n = {}^{n+1}C_2 \times {}^{n+1}C_2 = (\frac{n(n+1)}{2})^2 = (\sum n)^2 = \sum_{r=1}^{n} r^3 $$

13. Number of square of any size in $n \times n = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

14. Number of rectangle of any size in a rectangles of size $p \times n = {}^{p+1}C_2 \times {}^{n+1}C_2 = \frac{np(n+1)(p+1)}{4}$

15. Number of square of any size in a rectangles of size

$$ p \times n(p < n) = \sum_{r=1}^{p} (n + 1 - r)(p + 1 - r) $$

16. The number of regions into which a set of n coplanar lines in general position divide the plane is

$$ (\frac{n(n+1)}{2} + 1) $$

Restricted combinations

The concept of combination pertains to the methods of making selections. Represented as ${}^{n}C_r$, it signifies the total number of ways to select r elements from a set of n elements. Let us delve into an instance of a constrained combination. Suppose we are tasked with forming a team by selecting r players from a pool of n players, with two exceptional players always being automatically chosen. Consequently, the selection is narrowed down to (r-2) players from the remaining (n-2) candidates. The formula applicable to various scenarios of restricted combinations is outlined below.

Formula for Restricted Combination

1. The total ways to select r elements from a set of n elements are denoted as ${}^{n}C_r$.

2. When selecting r elements from n distinct items, with p specific elements always chosen, the expression is. ${}^{n-p}C_{r-p}$

3. When choosing r elements from n distinct items, and p particular elements are consistently excluded, the notation is. ${}^{n-p}C_r$

4. The method of selecting r distinct elements from n items, where k elements are always chosen, and p elements are systematically excluded, is represented as. ${}^{n-k-p}C_{r-k}$.

Ex. From a pool of 16 players, the task is to choose 11 individuals for a cricket team. Considering that the wicketkeeper and captain must be included in every selection, the question is how many ways this can be accomplished.

(a) ${}^{14}C_9$ (b) ${}^{16}C_{11}$ (C) ${}^{14}C_{11}$ (D) None of these

Sol. Given $n = 16$

$r = 11$

No. of players always included, $p = 2$

So, the number of ways $= {}^{n- -}C_{r-p} = {}^{14}C_9$

Hence, option 1 is the answer.