CIRCULAR PERMUTATIONS

We have observed that various items can be arranged in a linear sequence in multiple ways. However, when it comes to circular permutations, the specific positions of objects are not considered. Instead, only their relative positions hold significance. Consequently, arrangements involving 'r' of these objects are deemed equivalent or identical in terms of their relative positions. This characteristic of circular permutations influences the configurations of 'n' objects taken 'r' at a time.

$$^{n}Q_{r} = \frac{1}{r}\,^{n}P_{r} = \frac{n!}{r(n-r)!}$$

$$\text{Hence }^{n}Q_{n} = \frac{1}{n}n! = (n-1)!$$

In this scenario, clockwise and anticlockwise circular arrangements are considered distinct from each other.

When dealing with necklaces or garlands, no differentiation is made between the clockwise or anticlockwise arrangement of beads. This is because similar beads of the same color are arranged in both clockwise and anticlockwise directions. Consequently, in such cases, the total number of circular arrangements is halved. Thus, the permutation of n objects taken r at a time for a necklace is expressed as:

$$^{n}Q'_{r} = \frac{1}{2}\,^{n}Q_{r} = \frac{n!}{2r(n-r)!} \text{ and } ^{n}Q_{n}\,' = \frac{1}{2}\,^{n}Q_{n} = \frac{1}{2}(n-1)!$$

Ex. In how many ways can we arrange 6 different flowers in a circle? How many ways can we create a garland using these flowers?

Sol. The count of circular arrangements for 6 different flowers is (6 – 1)! = 120. When creating a garland, considering both clockwise and anticlockwise arrangements as equivalent, the number of ways to form the garland is also (6 – 1)! = 60.

Ex. In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls are together?

Sol. Seating 5 boys with one vacant seat between them can be done in 4! Ways.
Afterward, the 5 girls can be seated in the remaining 5 seats in 5! Ways.
Therefore, the total number of ways is 4! × 5.

Ex. The person invites a group of 10 friends to dinner, and they can be arranged in two different ways:

1. 5 on one round table and 5 on other round table,
2. 4 on one round table and 6 on other round table.

Determine the various ways in each scenario for arranging the guests.

Sol.

1. The ways to select 5 friends for the first table are $^{10}C_5$
   The remaining 5 friends are available for the second table.
   The total number of permutations of 5 guests at a round table is \ (4! \).
   Therefore, the total number of arrangements is (4!). $^{10}C_5 \times 4! \times 4!$
2. The number of ways of selection of 6 guests is $^{10}C_6$.
   The number of ways of permutations of 6 guests on round table is 5!
   The number of permutations of 4 guests on round table is 3!
   Therefore, total number of arrangements is: $^{10}C_5 . 5!\,3! = \frac{10!}{24}$