All possible selection

Selection from distinct objects:

The number of selections from n different objects, taken at least one

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots . + {}^nC_n = 2^n - 1$$

In other words, for every object we have two choice i.e. either selected or rejected in a particular group.

Total number of choices (all possible selections) $= 2.2.2 \ldots .$ n times $= 2^n$.

But this also included the case when none of then is selected and the number of such case = 1

Hence the number of selections, when at least one is selected $= 2^n - 1$

Selection from identical objects:

The number of selections of r objects out of n identical objects is 1.

Total number of selections of zero or more objects from n identical objects is n + 1.

The total number of selections of at least one out of $a_1 + a_2 + a_3 + \ldots + a_n$ objects, where a_1 are alike (of one kind), a_2 are alike (of second kind) and so on.... a_n are alike (of nth kind), is $[(a_1+1)(a_2 + 1)(a_3 + 1) \ldots (a_n + 1)] - 1$.

Selection when both identical and distinct objects are present:

The total number of selections, taking at least one from $a_1 + a_2 + a_3 + \ldots + a_n + k$ objects, where a_1 are identical (of the first kind), a_2 are identical (of the second kind), and so forth until a_n are identical (of the nth kind), and k are distinct, is given by the formula

$[(a_1 + 1)(a_2 + 1)(a_3 + 1) \ldots (a_n + 1)]\, 2^k - 1$.

Ex. Consider an individual who possesses 3 coins of 25 paise, 4 coins of 50 paise, and 2 coins of 1 rupee. The inquiry is then posed: In how many ways can he distribute none or some of these coins to a beggar? Additionally, determine the number of ways in the following scenarios:

1. Ensuring that he presents at least one coin of one rupee.

2. Ensuring that he presents at least one coin of each denomination.

Sol. Total number of ways of giving none or some coins is

$$(3 + 1)(4 + 1)(2 + 1) = 60 \text{ ways}$$

1. Number of ways of giving at least one coin of one rupee

$$= (3 + 1)(4 + 1) \times 2 = 40$$

2. Number of ways of giving at least one coin of each kind

$$= 3 \times 4 \times 2 = 24$$

Identical things of another type, r identical things of the third type, and n different things

The number of permutations of 'n' things, taken all at a time, when 'p' of them are the same and of one type, 'q' of them are the same and of the second type, 'r' of them are the same and of a third type, and the remaining n - (p + q + r) things are all different, is.

Ex. In how many ways can we arrange 3 red flowers, 4 yellow flowers, and 5 white flowers in a row? How many ways is this possible if the white flowers are to be separated in any arrangement? (Assuming flowers of the same color are identical).

Sol. Total we have 12 flowers 3 red, 4 yellow and 5 white.

Number of arrangements $= \frac{12!}{3!4!5!} = 27720$.

For the second part, first arrange 3 red & 4 yellow

This can be done in = 35 ways

Now select 5 places from among 8 places (including extremes) & put the white flowers there.

This can be done in ${}^8C_5 = 56$.

The number of ways for the 2^{nd} part $= 35 \times 56 = 1960$.

Ex. How many arrangements can be made using the digits 1, 2, 3, 4, 3, 2, 1 such that the odd digits are always in the odd positions?

Sol. There are four odd digits (1, 1, 3, 3) and four odd positions (first, third, fifth, and seventh). The odd digits can be arranged in 4! Ways at these positions. Then, at the remaining three places, the remaining three digits (2, 2, 4) can be arranged in 3! Ways.
The required number of numbers $= 6 \times 3 = 18$.

Ex. Determine the total number of 4-letter words that can be created using four letters from the word "PARALLELOPIPED."

Sol. Given letters are PPP, LLL, AA, EE, R, O, I, D.

Cases	No of ways of selection	No. of ways of arrangement	Total
All district	${}^8C_4$	${}^8C_4 \times 4!$	1680
2 a like, 2 district	$4C_1 \times {}^7C_2$	$4C1 \times {}^7C_2$	1008
2 a like, 2 other a like	${}^4C_2$	${}^4C_2 \times \frac{4!}{2!2!}$	36
3 a like, a district	${}^2C_1 \times {}^7C_1$	${}^2C_1 \times {}^7C_1 \times \frac{4!}{3!}$	56
		Total	2780

Number of divisors of N

Total Number Of Divisors

To find number of divisors of a given natural number greater than 1 we can write n as

$$n = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots\ldots p_n^{\alpha_n}$$

Where $p_1, p_2, \ldots, p_n$ are distinct prime numbers and $\alpha_1, \alpha_2, \ldots \alpha_n$ are positive integers.

Now any divisor of n will be of the form

$$d = p_1^{\beta_1} p_2^{\beta_2} p_3^{\beta_3} \ldots\ldots p_n^{\beta_n}$$

(Where $0 \le \beta_i \le \alpha_i, \beta_i \in I, \forall i = 1,2,3, \ldots, n$)

Here number of divisors will be equal to numbers of ways in which we can choose b_i' s which can be done in $(\alpha_1 + 1)(\alpha_2 + 1)\ldots(\alpha_n + 1)$ ways.

E.g. Let n = 360

$n = 2^3.3^2.5$

No. of divisors of $360 = (3 + 1)(2 + 1)(1 + 1) = 24$

Sum of all the divisors of n is given by

$$\left(\frac{p_1^{\alpha_1+1}-1}{p_1-1}\right) \cdot \left(\frac{p_1^{\alpha_2+1}-1}{p_2-1}\right) \cdot \left(\frac{p_1^{\alpha_3+1}-1}{p_3-1}\right) \ldots \left(\frac{p_n^{\alpha_n+1}-1}{p_n-1}\right)$$

As in above case, sum of all the divisors

$$= \left(\frac{2^4-1}{2-1}\right)\left(\frac{3^3-1}{3-1}\right)\left(\frac{5^2-1}{5-1}\right) = 1170$$

> The number of factors of a given natural number 'n' will be odd if and only if 'n' is a perfect square.