SERIES

A series is the summation of the terms of a sequence. When the sum extends to infinity, it is represented by S∞. The sum of the initial n terms in a sequence is known as a partial sum, denoted Sn. For instance, considering the sequence of positive odd integers 1, 3, 5... we express it as:

$$S_\infty = 1 + 3 + 5 + 7 + 9 + \cdots \text{ Infinite series}$$

$$S_5 = 1 + 3 + 5 + 7 + 9 = 255 \text{ th partial sum}$$

Ex. Find the 3rd and 5th partial sums of the sequence: 3, −6, 12, −24, 48...

Sol.
$$S_3 = 3 + (-6) + 12 = 9$$

$$S_5 = 3 + (-6) + 12 + (-24) + 48 = 33$$

Answer: $S_3 = 9; S_5 = 33$

If the general term is known, the series can be expressed using sigma (or summation) notation as follows:

$$S_\infty = \sum_{n=1}^{\infty} n^2 = 1^2 + 2^2 + 3^2 + \cdots \text{ Infinite series}$$

$$S_3 = \sum_{n=1}^{3} n^2 = 1^2 + 2^2 + 3^2 \text{ 3rd partial sum}$$

The symbol Σ (uppercase Greek letter sigma) is used to represent a series. The expressions above and below the sigma indicate the range of the index of summation, denoted by n. The lower number signifies the starting integer, and the upper value indicates the ending integer. The nth partial sum Sn can be expressed in sigma notation as follows:

$$S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

This is read as "the sum of ak as k goes from 1 to n." Replace n with ∞ to indicate an infinite sum.

Ex. $\sum_{k=1}^{5} (-3)^{k-1}$

Sol. $\sum_{k=1}^{5} (-3)^{k-1} = (-3)^{1-1} + (-3)^{2-1} + (-3)^{3-1} + (-3)^{4-1} + (-3)^{5-1}$

$$(-3)^0 + (-3)^1 + (-3)^2 + (-3)^3 + (-3)^4$$

$$1- 3 + 9 -27 +81 = 61$$

When dealing with sigma notation, the index is not always required to start at 1.

Ex. $\sum_{k=2}^{5} (-1)^k (3k)$

Sol. In this case, the index is represented by the variable k, spanning from 2 to 5.

$$\sum_{k=2}^{5} (-1)^k (3k) = (-1)^2 (3 \cdot 2) + (-1)^3 (3 \cdot 3) + (-1)^4 (3 \cdot 4) + (-1)^5 (3 \cdot 5)$$

$$6-9+12-15 = -6$$

Infinity is employed as the upper limit of a sum to signify an infinite series.

Ex. Write in expanded form: $\sum_{n=0}^{\infty} \frac{n}{n+1}$

Sol. In this instance, we commence with n = 0 and incorporate three dots to signify that this series extends indefinitely.

$$\sum_{n=0}^{\infty} \frac{n}{n+1} = \frac{0}{0+1} + \frac{1}{1+1} + \frac{2}{2+1} + \frac{3}{3+1} + \cdots$$

$$\frac{0}{1} + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

$$0 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

When expanding a series, ensure to substitute only the variable specified by the index.

Ex. Write in expanded form: $\sum_{i=1}^{\infty} (-1)^{i-1} x^{2i}$

Sol. $\sum_{i=1}^{\infty} (-1)^{i-1} x^{2i} = (-1)^{1-1} x^{2(1)} + (-1)^{2-1} x^{2(2)} + (-1)^{3-1} x^{2(3)} + \cdots$

$$(-1)^0 x^{2(1)} + (-1)^1 x^{2(2)} + (-1)^2 x^{2(3)} + \cdots$$

$$x^2 - x^4 + x^6 - \cdots$$

A sequence is a mathematical function with a domain that includes a set of natural numbers starting from 1. It can also be conceptualized as an ordered list.
Frequently, formulas are employed to express the nth term, or general term, of a sequence using the subscripted notation a_n.
A series is the accumulation of the terms in a sequence. The sum of the initial n terms is referred to as the nth partial sum and is denoted as S_n.
Sigma notation is employed to represent summations in a concise manner. The n^{th} partial sum, expressed in sigma notation, can be denoted

$$S_n = \sum_{k=1}^{n} a_k$$

The symbol Σ represents a summation, where the expression below indicates that the index k starts at 1 and iterates through the natural numbers, ending with the value n above.