Chapter 6

Sequences and Series

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INTRODUCTION

Sequences and series are fundamental concepts in arithmetic. A sequence is an ordered collection of elements where repetitions are allowed, while a series is the sum of all these elements. An arithmetic progression is a common example of a sequence and series.

In essence, a sequence is a collection of items or objects arranged in a sequential order.

A series can be broadly defined as the sum of all the terms in a sequence. However, there must be a clear relationship between all the terms of the sequence.

A deeper understanding of the fundamentals can be gained by solving problems that involve the application of formulas. Sequences share similarities with sets, but the key distinction lies in the fact that in a sequence, individual terms may occur repeatedly in different positions. The length of a sequence is determined by the number of terms it contains, and a sequence can be either finite or infinite.

SEQUENCE AND SERIES DEFINITION

A sequence is a systematic arrangement of objects or a set of numbers following a specific order and governed by a rule. If (a_1, a_2, a_3, a_4, ldots) represent the terms of a sequence, then 1, 2, 3, 4, etc., denote the positions of the terms. The definition of a sequence can be established based on the number of terms.

Either finite sequence or infinite sequence.

$a_1, a_2, a_3, a_4 \ldots$ is a sequence, then the corresponding series is given by

$$S_N = a_1 + a_2 + a_3 + \cdots + a_N$$

The nature of the series, whether finite or infinite, is determined by whether the underlying sequence is finite or infinite.

Types of Sequence and Series

Some frequently encountered examples of sequences include:

1. Arithmetic Sequences
2. Geometric Sequences
3. Harmonic Sequences
4. Fibonacci Numbers

1. **Arithmetic Sequences**

   An arithmetic sequence is a sequence in which each term is generated by adding or subtracting a constant value to the preceding number.

2. **Geometric Sequences**

   A geometric sequence is a sequence in which each term is derived by multiplying or dividing a constant value with the preceding number.

3. Harmonic Sequences

A set of numbers is considered to be in harmonic sequence if the reciprocals of all the elements in the sequence create an arithmetic sequence.

4. Fibonacci Numbers

Fibonacci numbers constitute a captivating sequence in which each element is derived by adding the two preceding elements. The sequence begins with 0 and 1, and it is defined as follows:

$$F_0 = 0, F_1 = 1, \text{ and } F_m = Fn_{-1} + Fn_{-2}.$$

SEQUENCES

A sequence is a function defined on a set of consecutive natural numbers starting from 1. For instance, the infinite sequence given by the equation with the domain {1, 2, 3 ...} is an example:

$$a(n) = 5n - 3 \text{ or } a_n = 5n - 3$$

The values in the range of this function are referred to as the terms of the sequence. It is typical to express the nth term, or the general term of a sequence, using subscripted notation an, read as "a sub n." The terms can be determined by substitution as follows:

General term: $a_n = 5n - 3$

First term (n = 1):	$a_1 = 5(1) - 3 = 2$
Second term (n = 2):	$a_2 = 5(2) - 3 = 7$
Third term (n=3):	$a_3 - 5(3) - 3 = 12$
Fourth term (n = 4):	$a_3 = 5(4) - 3 = 17$
Fifth term (n = 5):	$a_3 = 5(5) - 3 = 22$

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Ex. Find the first five terms and the 100th term of the sequence given its general term:$a_n = \frac{n(n-1)}{2}$

Sol. To determine the initial five terms, substitute the values 1, 2, 3, 4, and 5 for (n) in the given expression and simplify.

$$a_1 = \frac{1(1-1)}{2} = \frac{1(0)}{2} = \frac{0}{2} = 0$$
$$a_2 = \frac{2(2-1)}{2} = \frac{2(1)}{2} = \frac{2}{2} = 1$$
$$a_3 = \frac{3(3-1)}{2} = \frac{3(2)}{2} = \frac{6}{2} = 3$$
$$a_4 = \frac{4(4-1)}{2} = \frac{4(3)}{2} = \frac{12}{2} = 6$$
$$a_5 = \frac{5(5-1)}{2} = \frac{5(4)}{2} = \frac{20}{2} = 10$$

Utilize (n = 100) to ascertain the 100th term within the sequence.

$$a_{100} = \frac{100(100-1)}{2} = \frac{100(99)}{2} = \frac{9,900}{2} = 4,950$$

Answer: First five terms: 0, 1, 3, 6, 10; a_{100}=4,950

Occasionally, the general term of a sequence may exhibit alternating signs and incorporate a variable other than (n).

Ex. Determine the initial 5 terms of the sequence an $= (-1)^n x^{n+1}$

Sol. Here, it is important to substitute the first 5 natural numbers for (n) and not (x).

$$a_1 = (-1)^1 x^{1+1} = -x^2$$
$$a_2 = (-1)^2 x^{2+1} = x^3$$
$$a_3 = (-1)^3 x^{3+1} = -x^4$$
$$a_4 = (-1)^4 x^{4+1} = x^5$$
$$a_5 = (-1)^5 x^{5+1} = -x^6$$

Answer: $-X^2, X^3, -x^4, X^5, -X^6$

Try this! Find the first 5 terms of the sequence: an $= (-1)^{n+1} 2^n$

Answer: 2, −4, 8, −16, 32.

An intriguing example is the Fibonacci sequence. The initial two numbers in the Fibonacci sequence are 1, and each subsequent term is the sum of the preceding two. Hence, the general term is expressed in relation to the previous two as follows:

$$F_n = F_{n-2} + F_{n-1}$$

Here $F_1 = 1, F_2 = 1, \text{and } n > 2.$

A formula that characterizes a sequence based on its preceding terms is known as a recurrence relation.

Ex. Determine the initial 7 numbers in the Fibonacci sequence.

Sol. Given that $F_1=1$and $F_2=1$, use the recurrence relation $F_n=F_{n-2}+F_{n-1}$where n is an integer starting with n=3 to find the next 5 terms:

$$F_3 = F_{3-2} + F_{3-1} = F_1 + F_2 = 1 + 1 = 2$$
$$F_4 = F_{4-2} + F_{4-1} = F_2 + F_3 = 1 + 2 = 3$$
$$F_5 = F_{5-2} + F_{5-1} = F_3 + F_4 = 2 + 3 = 5$$
$$F_6 = F_{6-2} + F_{6-1} = F_4 + F_5 = 3 + 5 = 8$$
$$F_7 = F_{7-2} + F_{7-1} = F_5 + F_6 = 5 + 8 = 13$$

Answer: 1,1,2,3,5,8,13