Relation Between AM & GM

For two positive real numbers, a and b

$$A = \frac{a+b}{2}, G = \sqrt{ab}$$

$$A > G \text{ if } a \neq b \quad \ldots \text{(i)}$$

$$A = G \text{ if } a = b \quad \ldots \text{(ii)}$$

So, combining (i) & (ii),

We have A ≥ G, and equality holds when a = b.

If $a_1, a_2, a_3 \ldots a_n$ are n positive numbers,

Then, above discussion leads to the result that,

$$\frac{a_1+a_2+a_3+\cdots.+a_n}{n} \geq \sqrt[n]{a_1a_2a_3\ldots.a_n}$$

Ex. There are n arithmetic means between 1 and 31, and the 7th mean to the $n-1^{th}$ mean forms a ratio of º 5 : 9. Find n ?

Sol. Let, d and A_j denote the common difference and j^{th} Arithmetic mean respectively;

$$\boldsymbol{d = \frac{31-1}{n+1} = \frac{30}{n+1}}$$

$$\boldsymbol{A_7 = 1 + 7\frac{30}{n+1} = 1 + \frac{210}{n+1}}$$

$$A_{n-1} = 1 + (n-1)\frac{30}{n+1}$$

$$\frac{A_7}{A_{n-1}} = \frac{5}{9}$$

$$9 + \frac{1890}{n+1} = 5 + \frac{150(n-1)}{n+1}$$

$$146n = 2044$$

$$n = 14$$

Ex. If one arithmetic mean (A.M.), A, and two geometric means (G.M.s) p and q are inserted between any two given numbers, then it can be shown that $\mathbf{p^3 + q^3 = 2}$ Apq.

Sol. Let the two given numbers be, a and b;

Then, $2A = a + b \ldots \text{(i)}$

a, p, q, b are in G.P.

$$p^2 = aq \text{ and } q^2 = bp$$

$$p^3 = apq \text{ and } q^3 = bpq$$

$$p^3 + q^3 = (a+b)pq = 2Apq$$

Special Series

Sigma (S) notation: S indicates sum i.e., $\sum_{i=1}^{n} i = \sum n = 1 + 2 + 3 + \cdots \ldots. + n$

$$\sum_{l=1}^{n} \frac{i+1}{i+2} = \frac{1+1}{1+2} + \frac{2+1}{2+2} + \frac{3+1}{3+2} + \cdots. + \frac{n+1}{n+2}$$

$$\sum_{1=1}^{m} a = a + a + \cdots \ldots + a \text{ m times } = am \text{ where a is constant}$$

$$\sum_{l=1}^{m} ai = a\sum_{l=1}^{m} i = a(1 + 2 + \cdots. + m)$$

$$\sum_{l=1}^{m}(i^3 - 2i^2 + i) = \sum_{l=1}^{m} i^3 - 2\sum_{l=1}^{m} i^2 + \sum_{l=1}^{m} i$$

Important Results

1. Sum of the first n natural numbers.

$$\Sigma n = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

2. Sum of the squares of the first n natural numbers.

$$Sn^2 = 1^2 + 2^2 + \cdots..+n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of the cubes of the first n natural numbers.

$$Sn^3 = 1^3 + 2^3 + 3^3 + \cdots..+n^3 = (\frac{n(n+1)}{2})^2 = (\Sigma n)^2$$

4. Sum of the first n terms of a sequence $T_n = a^3 + bn^2 + cn + d$

$$S_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + dn$$

Ex. Calculate the sum of the series 3.5 + 6.8 + 9.11 + ... up to n terms.

Sol. n^{th} term of 3,6,9, … is 3n

N^{th} term of 5,8,11, … is (3n + 2)

$$T_n = 3n(3n+2) = 9n^2 + 6n$$
$$S_n = 9Sn^2 + 6Sn$$
$$= \frac{9n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2}$$
$$\frac{3}{2}n(n+1)[2n+1+2]$$
$$= \frac{3n(n+1)(2n+3)}{2}$$

Arithmetico-geometric series (A.G. S.)

$$n^{th} \text{ term of A.G.. S. } = (n^{th} \text{ term of an A.P. }) \times (n^{th} \text{ term of A G.P. })$$

If $a, (a+d), (a+2d) + \cdots$ be an a.p. &$b, br, br^2 + \cdots..$ be a g.p.

Then $ab + (a+d)br + (a+2d)b^2 + \cdots$. Is The Corresponding A.G.S.

$$T_n \text{ of A.G.S. } = (T_n \text{ of A.P. }) \times (T_n \text{ of G.P. })$$