# ARITHMETIC PROGRESSION:

An arithmetic sequence, also known as an arithmetic progression, is a sequence of numbers in which each succeeding number is obtained by adding a constant difference, denoted as "d," to the preceding number.

$$a_n = a_{n-1} + d \quad \text{Arithmetic Sequence}$$

Because $a_n - a_{n-1} = d$, the constant d is called the **common difference**.

**For example,**

The sequence of positive odd integers forms an arithmetic sequence, starting with 1 and having a common difference of 2 between any two consecutive terms:

$$1,3,5,7,9 \ldots \ldots$$

We can construct the general term $a_n = a_{n-1} + 2$

Where,

$$a_1 = 1$$
$$a_2 = a_1 + 2 = 1 + 2 = 3$$
$$a_3 = a_2 + 2 = 3 + 2 = 5$$
$$a_4 = a_3 + 2 = 5 + 2 = 7$$
$$a_5 = a_4 + 2 = 7 + 2 = 9$$

.

In general, for an arithmetic sequence with the initial term ($a_1$) and a common difference (d), the terms of the sequence can be expressed as follows:

$$a_2 = a_1 + d$$
$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$$
$$a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$$
$$a_5 = a_4 + d = (a_1 + 3d) + d = a_1 + 4d$$

.

This implies that any arithmetic sequence can be represented using its initial term, common difference, and index in the following manner:

$$a_n = a_1 + (n-1)d \text{ Arithmetic Sequence}$$

Indeed, any general term that follows a linear expression in terms of n defines an arithmetic sequence.

**Ex.** Determine an expression for the general term of the provided arithmetic sequence and use it to compute the 100th term: 7, 10, 13, 16, 19…

**Sol.** Start by determining the common difference, (d = 10 - 7 = 3). It is evident that the difference between consecutive terms is 3, confirming that the sequence is an arithmetic progression with ($a_1 = 7$) and (d = 3).

$$a_n = a_1 + (n-1)d$$
$$= 7 + (n-1) \cdot 3 = 7 + 3n - 3$$
$$= 3n + 4$$

Hence, we can express the general term as ($a_n = 3n + 4$). Take a moment to confirm that this equation accurately represents the provided sequence. Utilize this equation to determine the value of the 100th term:

$$a_{100} = 3(100) + 4 = 304$$

**Answer:** $a_n = 3n + 4; a100 = 304$

The common difference of an arithmetic sequence may be negative.

**Ex.** Find an equation for the general term of the given arithmetic sequence and use it to calculate its $75^{th}$ term: 6, 4, 2, 0,−2

**Sol.** Begin by finding the common difference, $d = 4 - 6 = -2$

Next find the formula for the general term, here $a_1 = 6$and $d = -2$.

$$a_n = a_1 + (n-1)d$$
$$6 + (n-1) \cdot (-2) = 6 - 2n + 2$$
$$8 - 2n$$

Therefore, $a_n = 8 - 2n$ and the $75^{th}$ term can be calculated as follows:

$$a_{75} = 8 - 2(75) = 8 - 150 = -142$$

**Answer:** $a_n = 8 - 2n; a_{75} = -142$

The terms between given terms of an arithmetic sequence are called **arithmetic means.**

**Ex.** Determine all the terms between ($a_1$ = – 8) and ($a_7$ = 10) in an arithmetic sequence, essentially identifying all the arithmetic means between the 1st and $7^{th}$ terms.

**Sol.** Start by determining the common difference, (d). In this scenario, we have information about the first and seventh terms:

$$a_n = a_1 + (n-1)d \text{ use } n = 7$$
$$a_7 = a_1 + (7-1)d$$
$$a_7 = a_1 + 6d$$

Replace ($a_1$ = –8) and ($a_7$ = 10) in the provided equation, then solve for the common difference (d).

$$10 = -8 + 6d$$
$$18 = 6d$$
$$3 = d$$

Afterwards, utilize the initial term ($a_1$ = –8) and the common difference (d = 3) to derive an expression for the $(n)^{th}$ term of the sequence.

$$a_n = -8 + (n-1) \cdot 3$$
$$= -8 + 3n - 3$$
$$= -11 + 3n$$

With $a_n = 3n - 11$, where n is a positive integer, find the missing terms.

$$a_1 = 3(1) - 11 = 3 - 11 = -8$$
$$a_2 = 3(2) - 11 = 6 - 11 = -5$$
$$a_3 = 3(3) - 11 = 9 - 11 = -2$$
$$a_4 = 3(4) - 11 = 12 - 11 = 1$$
$$a_5 = 3(5) - 11 = 15 - 11 = 4$$
$$a_6 = 3(6) - 11 = 18 - 11 = 7$$
$$a_7 = 3(7) - 11 = 21 - 11 = 10$$

**Answer:** $-5, -2, 1, 4, 7$

In some cases, the first term of an arithmetic sequence may not be given.

**Ex.** Determine the general term of an arithmetic sequence given that $a_3$ = –1 and $a_{10}$ = 48.

**Sol.** To find a formula for the general term, we need values for ($a_1$) and (d). We can set up a system of linear equations using the provided information and solve for these variables.

$$a_n = ai + (n-1)d:$$
$$a_3 = a_1 + (3-1)d$$
$$-1 = a_1 + 2d \quad (\text{Use } a_3 = -1)$$
$$a_{10} = a_1 + (10-1)d$$
$$48 = a_1 + 9d \quad (\text{Use } a_{10} = 48)$$

Remove ($a_1$) by multiplying the first equation by (-1) and then adding the result to the second equation.

$$-1 = (a_1 + 2d) \times (-1)$$
$$1 = -a_1 - 2d \quad \ldots (i)$$
$$48 = a_1 + 9d$$
$$48 = a_1 + 9d \quad \ldots (ii)$$

Adding equation (i) and (ii)

$$49 = 7d$$
$$7 = d$$

Substitute d = 7 into $-1 = a_1 + 2d$ to find $a_1$.

$$-1 = a_1 + 2(7)$$
$$-1 = a_1 + 14$$
$$-15 = a_1$$

Next, use the first term $a_1=-15$ and the common difference $d=7$ to find a formula for the general term.

$$a_n = a_1 + (n-1)d$$
$$-15 + (n-1)\cdot 7$$
$$-15 + 7n - 7$$
$$-22 + 7n$$

**Answer:** $a_n = 7n - 22$

## Arithmetic Series:

An arithmetic series is the sum of the terms of an arithmetic sequence. For instance, the sum of the first 5 terms of the sequence defined by ($a_n = 2n - 1$) is given by:

$$S_5 = \sum_{n=1}^{5}(2n-1)$$
$$= [2(1)-1] + [2(2)-1] + [2(3)-1] + [2(4)-1] + [2(5)-1]$$
$$= 1 + 3 + 5 + 7 + 9 = 25$$

Summing 5 positive odd integers, as demonstrated earlier, is manageable. However, envision adding the first 100 positive odd integers; this would become quite tedious. Consequently, we will now derive a formula that can be employed to compute the sum of the first n terms, denoted as (Sn), for any arithmetic sequence. In general,

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + a_n$$

Writing this series in reverse we have,

$$S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + a_1$$

Adding these two equations together, the terms involving (d) sum to zero, yielding (n) factors of ($a_1 + a_n$):

$$2S_n = (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_n + a_1)$$
$$2S_n = n(a_1 + a_n)2)$$

Dividing both sides by 2 gives us the formula for the nth partial sum of an arithmetic sequence:

$$S_n = \frac{n(a_1+a_n)}{2}$$

Apply this formula to compute the sum of the first 100 terms of the sequence given by an=2n−1, where a1=1 and $a_{100}=199$.

$$S_{100} = \frac{100(a_1+a_{100})}{2} = \frac{100(1+199)}{2} = 10{,}000$$

**Ex.** Determine the sum of the initial 50 terms in the provided sequence: 4, 9, 14, 19, 24, ...

**Sol.** Establish whether a common difference exists among the provided terms. d = 9 – 4 = 5

Observe that the gap between consecutive terms is 5. The sequence is, indeed, an arithmetic progression, and we can express it as

$$a_n = a_1 + (n-1)d$$
$$= 4 + (n-1)\cdot 5$$
$$= 4 + 5n - 55n - 1$$

Hence, the general term is given by (an = 5n – 1). To find the sum of the first 50 terms of this sequence, we need the values of the $1^{st}$ and $50^{th}$ terms:

$$a_1 = 4$$
$$a_{50} = 5(50) - 1 = 249$$

Now, utilize the formula to find the $50^{th}$ partial sum of the provided arithmetic sequence.

$$S_n = \frac{n(a_1+a_n)}{2}$$
$$S_{50} = \frac{50\cdot(a_1+a_{50})}{2}$$
$$\frac{50(4+249)}{2} = 25(253) = 6{,}325$$

**Answer:** $S_{50}=6{,}325$

**Ex.** Evaluate: $\sum_{n=1}^{35}(10-4n)$

**Sol.** Here, the task is to calculate the sum of the initial 35 terms of an arithmetic sequence characterized by the general term an $= 10 - 4n$. This involves determining the 1st and 35th terms.

$$a_1 = 10 - 4(1) = 6$$

$$a_{35} = 10 - 4(35) = -130$$

Next use the formula to determine the 35th partial sum.

$$S_n = \frac{n(a_1+a_n)}{2}$$

$$S_{35} = \frac{35\cdot(a_1+a_{35})}{2}$$

$$= \frac{35[6+(-130)]}{2} = \frac{35(-124)}{2} = -2{,}170$$

**Answer:** $-2{,}170$

An arithmetic sequence is a sequence in which the difference (d) between successive terms is constant.

The general term of an arithmetic sequence can be expressed in terms of its first term ($a_1$), common difference (d), and index (n) as follows:

$$a_n = a_1 + (n-1)d$$

An arithmetic series is the summation of the terms of an arithmetic sequence.

The nth partial sum of an arithmetic sequence can be determined using the first and last terms with the formula:

$$S_n = \frac{n(a_1+a_n)}{2}$$