Chapter 5

Complex Number

INTRODUCTION

To address equations such as $x^2 + 1 = 0$, it became necessary to broaden the scope beyond the realm of real numbers, leading to the development of the complex number system. The exploration of complex numbers holds significant implications across various domains in science and engineering. This discussion will delve into fundamental concepts associated with complex numbers.

SQUARE ROOT OF NEGATIVE NUMBERS

$\sqrt{-1}$ is defined as i (Pronounced as iota)

i.e., $i = \sqrt{-1} \Rightarrow i^2 = -1 \Rightarrow i^3 = -i \Rightarrow i^4 = 1$

Infect, if a is a positive real number then

$$\sqrt{-a} = \sqrt{a}\sqrt{-1} = \sqrt{a}i$$

1. For any integer m,
   $i^{4m} = 1, i^{4m+1} = i, i^{4m+2} = i^2 = -1 i^{4m+3} = i^3 = -i$
2. The total of consecutive fourth powers
   $$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, n \in \mathbb{Z}$$
3. $\frac{1}{i} = -i$ and $-\frac{1}{i} = i$

Ex. Determine the numerical result of i^{39}.

Sol. $i^{39} = i^{4\times9+3} = i^3 = -i$

Ex. Calculate the numerical value of

$$y = i + i^2 + i^3 + \cdots..+i^{100} + i^{101}$$

Sol. We observe that

$$y = (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) +$$
$$\ldots.+(i^{97} + i^{98} + i^{99} + i^{100}) + i^{101}$$
$$= (0 + 0 + \cdots.) + i^{101} = i^{4\times25+1} = i.$$

Ex. If x = (1 + i), then determine the value of $(x - 1)^4 + 4$.

Sol.
$$x = 1 + i \Rightarrow x - 1 = i$$
$$\text{Now, } (x - 1)^4 = 1$$
$$\therefore (x - 1)^4 + 4 = 5$$

For positive real numbers a, and b, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ this conclusion remains valid when either $a > 0, b < 0$, or $a < 0, b > 0$. If it is true, then $i^2 = i.i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$
Which is a contradiction.
Thus if $a < 0, b < 0$, then $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$. Also, if any of a or b is 0, then $\sqrt{a} \times \sqrt{b} = \sqrt{ab} = 0$.