PROPERTIES OF ARGUMENT OF A COMPLEX NUMBER

The following properties regarding argument of complex numbers hold good:

1. $\arg(\overline{z}) = -\arg(z)$, provided z is not negative purely real number.

2. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2k\pi (k = 0,1 \text{ or } -1)$

In general $\arg(z_1 z_2 z_3 \ldots z_n) = \arg(z_1) + \arg(z_2) + \arg(z_3) + \cdots \ldots + \arg(z_n) + 2k\pi, k \in I$.

3. $\arg(\frac{z_1}{z_2}) = \arg(z_1) - \arg(z_2) + 2k\pi; k = 0 \text{ or } 1 \text{ or } -1$

4. $\arg(\frac{z}{\overline{z}}) = 2\arg(z) + 2k\pi; (k = 0,1 \text{ or } -1)$

5. $\arg(z^n) = n\arg(z) + 2k\pi$; k is integer depending on ' n '

6. If $\arg(z) = 0$ or π, then z is purely real number.

7. If $\arg(z) = \frac{\pi}{2}$ or $-\frac{\pi}{2}$, then z is purely imaginary number.

Ex. If $\arg(z_1) = \frac{2\pi}{3}$ and $\arg(z_2) =$ then find the principal value of $\arg(z_1 z_2)$ and also find the quadrant of $z_1 z_2$.

Sol. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2k\pi$

$$= \frac{2\pi}{3} + \frac{\pi}{2} + 2k\pi$$

$$= \frac{7\pi}{6} + 2k\pi$$

To bring the argument in the principal range k = –1

$\arg(z_1 z_2) = \frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$ which shows that $z_1 z_2$ lies in IIIrd quadrant.

Ex. If $z = \frac{(1+i)(1-i)}{(2-i)}$, find the principal argument of z and also write the polar form of z.

Sol. $z = \frac{(1+i)(1-i)}{(2-i)} = \frac{2(2+i)}{(2^2-i^2)} = \frac{2}{5}(2 + i) = \frac{4}{5} + \frac{i2}{5}$

Principal argument of $z = \tan^{-1}(\frac{1}{2})$

$$|z| = \sqrt{\frac{16}{25} + \frac{4}{25}} = \sqrt{\frac{20}{25}} = \sqrt{\frac{4}{5}}$$

Polar form $z = \sqrt{\frac{4}{5}}(\cos\theta + i\sin\theta)$, where $\theta = \tan^{-1}\frac{1}{2}$.

Ex. Find the complex number z if $z\overline{z} = 2$ and $z + \overline{z} = 2$.

Sol. Let, $z = x + iy$

$z\overline{z} = 2 \Rightarrow (x + iy)(x - iy) = 2 \quad \Rightarrow \quad x^2 + y^2 = 2 \ldots (i)$

Also $\quad z + \overline{z} = x + iy + x - iy = 2x = 2 \quad \Rightarrow \quad x = 1 \ldots (ii)$

By (i) and (ii), we have

$$x = 1, y = \pm 1$$

Hence, $\quad z = 1 + i \text{ or } 1 - i$