DIFFERENT FORMS OF COMPLEX NUMBERS

A complex number can be expressed in three distinct forms.

1. Cartesian form

The expression $z = x + iy$, where $x \in R$ and $y \in R$, is referred to as the Cartesian form. It has been previously mentioned that $x = \text{Re}(z)$ and $y = \text{Im}(z)$.

Applications

This particular representation is frequently utilized for:

(a) The spatial representation of complex numbers on the Argand plane

(b) Establishing the locus of complex numbers

2. Polar form (r, θ)

$$OP = r,$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = x + iy$$
$$r\cos\theta + ir\sin\theta$$
$$r(\cos\theta + i\sin\theta)$$

This is referred to as the Trigonometric (or Polar) form of a complex number. In this context, it is important to consider the principal value of θ. for arbitrary values of the argument,

$z = r[\cos(2n\pi + \theta) + i\sin(2n\pi + \theta)]$ (Where n is an integer)

At time, $\cos\theta + i\sin\theta$, is denoted as c is (θ).

Euler's Form

In accordance with Euler's Theorem, $e^{i\theta} = \cos\theta + i\sin\theta$. Consequently

$z = r(\cos\theta + i\sin\theta)$ Can be written as $z = re^{i\theta}$ which is termed the exponential form of a complex number.

Replacing θ by $-\theta$ in $e^{i\theta}$

We obtain $e^{-i\theta} = \cos\theta - i\sin\theta$

Hence $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{x}{\sqrt{x^2+y^2}}$

And $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i} = \frac{y}{\sqrt{x^2+y^2}}$

Ex. Express the given complex numbers in both polar and exponential forms.

1. $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ 2. $1 - i$

Sol. 1. Here, $r\cos\theta = -\frac{1}{2}, r\sin\theta = -\frac{\sqrt{3}}{2}$

Squaring and adding $r^2\cos^2\theta + r^2\sin^2\theta = \frac{1}{4} + \frac{3}{4} = 1$

$\therefore r = 1(-\text{ ve value is rejected })$

Dividing we get $\tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$

Since $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ lies in third quadrant

Principal argument $= \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

Polar form $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ is $1(\cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3}))$

And Euler's form is $1.e^{-\frac{2\pi}{3}i}$

2. Here $r\cos\theta = 1$ and $r\text{in}\,\theta = -1$

$\therefore$ $r = \sqrt{2}, \tan\theta = -1 = \tan(-\frac{\pi}{4})$

Since (1,-1) lies in IV ^{e} quadrant, principal value of q is $-\frac{\pi}{4}$

Polar form of $1-i$ is $\sqrt{2}(\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4}))=\sqrt{2}\times e^{-\frac{i\pi}{4}}$

$$1=e^{0}, i=e^{\frac{i\pi}{2}}, -i=e^{-\frac{i\pi}{2}}, -1=e^{i\pi}$$

$$\log i=\log e^{\frac{i\pi}{2}}=i\frac{\pi}{2}; \log(\log i)=\log(i\frac{\pi}{2})=\log i+\log\frac{\pi}{2}=i\frac{\pi}{2}+\log\frac{\pi}{2}$$