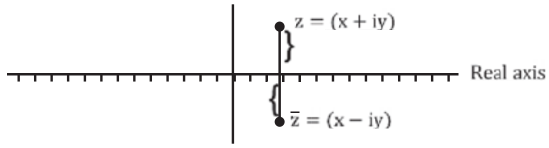
Conjugate of a Complex Number

The mirror reflection of a complex number $z = x + iy$ across the real axis is referred to as the conjugate of z. This point is represented by $\bar{z} = x - iy$.



Properties of Conjugate of a Complex Number

1. $\overline{(\bar{z})} = z$

2. $|z| = |\bar{z}| = |-z| = |-\bar{z}| = |iz| = |i\bar{z}|$

3. $z\bar{z} = |z|^2 = (\text{Re}(z))^2 + (\text{lm}(z))^2$

4. $\text{Re}(z) = \frac{z+\bar{z}}{2}, \text{Im}(z) = \frac{z-\bar{z}}{2i}$

If z is purely real number, then Im (z) = 0 $\Rightarrow$ $z = \bar{z}$

If z is purely imaginary, then Re (z) = 0 $\Rightarrow$ $z + \bar{z} = 0$ or $z = -\bar{z}$

5. $(\overline{z_1 + z_2}) = \bar{z}_1 + \bar{z}_2$

In general $\overline{z_1 + z_2 + \cdots \dots + z_n} = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 + \cdots \dots + \bar{z}_n$

6. $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$ or in general $\overline{z_1 \cdot z_2 \dots \dots z_n} = \bar{z}_1 \cdot \bar{z}_2 \dots \dots \bar{z}_n$

And also we may write that $(\overline{z^n}) = (\bar{z})^n$

7. If $z_2 \neq 0$, then $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

8. $\arg(z) + \arg(\bar{z}) = 2k\pi, k \in Z$

Ex. If $z_1 = \bar{z}_1$ and $z_2 = -\bar{z}_2$ and z_1 and z_2 both are non-zero complex numbers, then find the sum of all possible principal values of $\arg(z_1)$ and $\arg(z_2)$.

Sol. If $z_1 = \bar{z}_1$ then z_1 is purely real number and in this case $\arg(z_1) = 0$ or π.

Similarly if $z_2 = -\bar{z}_2$, then z_2 is purely imaginary number and in this case $\arg(z_2) = \frac{\pi}{2}, -\frac{\pi}{2}$

Hence sum $= 0 + \pi + \frac{\pi}{2} - \frac{\pi}{2} = \pi$.