TRANSFORMATION OF EQUATIONS

Let's examine a polynomial equation.

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots \ldots \ldots \ldots + a_n = 0$ ... (1)

The root of the given equation is denoted as x. To derive a new equation with a general root y, where $y = f(x)$, substitute x with g(x) in (i), where $x = g(y)$ is obtained from $y = f(x)$, subject to a certain condition. Let's illustrate this process.

Ex. If α, β, and γ represent the roots of the equation $x^3 + x^2 + x + 9 = 0$, determine the equation whose roots are $\frac{\alpha+1}{\alpha-1}, \frac{\beta+1}{\beta-1}, \frac{\gamma+1}{\gamma-1}$

Sol. Let $y = \frac{x+1}{x-1} \Rightarrow x = \frac{y+1}{y-1}$

Hence to obtained new equation replace x by $\frac{x+1}{x-1}$ which is given by.

$$\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x+1}{x-1}\right) + 9 = 0$$

$$(x+1)^3 + (x+1)^2(x-1) + (x+1)(x-1)^2 + 9(x-1)^3 = 0$$

$$\Rightarrow 12x^3 - 24x^2 + 28x - 8 = 0$$

$$\Rightarrow 3x^3 - 6x^2 + 7x - 2 = 0$$