QUADRATIC EXPRESSION IN TWO VARIABLES

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ Is a quadratic expression in the variables x and y. The expression can be factored into two linear rational factors if

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Ex. Determine the value of k for which the expression $x^2 + 4xy + y^2 + 4x + 2y + k = 0$ can be factored into two linear expressions.

Sol. Comparing with, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we have

$$a = 1, b = 1, h = 2, g = 2, f = 1, c = k$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\Rightarrow 1 \times 1 \times k + 2 \times 1 \times 2 \times 2 - 1 \times 1 - 1 \times 4 - k \times 4 = 0$$

$$\Rightarrow 3k = 3$$

$$\Rightarrow k = 1.$$