QUADRATIC EXPRESSION

An expression of the form $ax^2 + bx + c$, where $0 \neq a, b, c \in R$ is called a quadratic expression or quadratic polynomial in independent variable x.

Let $y = f(x) = ax^2 + bx + c = a[x^2 + \frac{b}{a}x + \frac{c}{a}] = a[x^2 - (\alpha + \beta)x + \alpha\beta] = [a(x - \alpha)(x - \beta)]$

α and β represent the solutions of the equation $ax^2 + bx + c = 0$.

Here, α and β denote the roots of the associated quadratic equation $ax^2 + bx + c = 0$.

$$y = f(x) = a[x^2 + 2x \cdot \frac{b}{2a} + (\frac{b}{2a})^2] - \frac{b^2}{4a} + c$$

$$= a[(x + \frac{b}{2a})^2] - \frac{b^2-4ac}{4a}$$

$$(y + \frac{D}{4a}) = a[(x + \frac{b}{2a})^2]; D = b^2 - 4ac$$

Which represent a parabola with vertex at $(\frac{-b}{2a}, \frac{-D}{4a})$

Range of the Quadratic Function

Let $f(x) = ax^2 + bx + c, a \neq 0$

A. Range when $x \in R$

$$\text{If } a > 0 \text{ then } f(x) \in [-\frac{D}{4a}, \infty)$$

$$\text{If } a < 0, \text{ then } f(x) \in (-\infty, \frac{-D}{4a})$$

B. Range in restricted domain

Case I: $(x_1 < -\frac{b}{2a} < x_2)$

For $f(x) = ax^2 + bx + c, x \in [x_1, x_2]$ find $f(x_1), f(x_2)$ and $f(-\frac{b}{2a})$

If the least value of f(x) among $f(x_1), f(x_2)$ and $f\left(-\frac{b}{2a}\right)$ is m and greatest value is M, then the range off(x) is [m, M].

Case II: $-\frac{b}{2a} \notin (x_1, x_2)$

For the function $f(x) = ax^2 + bx + c$, where $x \in [x_1, x_2]$, determine $f(x_1)$ and $f(x_2)$. If the minimum of $f(x_1)$ and $f(x_2)$ is denoted as m, and the maximum as M, then the range of f(x) is [m, M].