PROPERTIES RELATED TO NATURE OF ROOTS OF QUADRATIC EQUATION

Consider D_1 and D_2 as the discriminants of two provided quadratic equations, $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$, respectively.

1. If $D_1 + D_2 \geq 0$
Then at least one of $D_1 \& D_2$ is ≥ 0
At least one of the equations has real roots.

2. If $D_1 + D_2 < 0$, then at least one of D_1 and $D_2 < 0$
At least one of the provided equations has roots in the imaginary number domain.
For example, let us consider two quadratic equations $x^2 + 4x + 3 = 0$ and $x^2 + 2x + 4 = 0$.

Here $D_1 = 16 - 12 = 4 \& D_2 = 4 - 16 = -12$
$$D_1 + D_2 = 4 - 12 = -8 < 0$$
At least one of the provided equations exhibits roots in the imaginary domain.
Here $x^2 + 2x + 4 = 0$ has imaginary roots.

3. If $D_1D_2 < 0$
Then the equation $(a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2) = 0$ has two real roots.
In the above example we observe that $D_1D_2 = 4 \times -12 = -48 < 0$.
Hence the equation $(x^2 + 4x + 3)(x^2 + 2x + 4) = 0$ has two real and two complex roots.

4. If $D_1D_2 > 0$, then
Either $D_1 > 0$ or $D_2 > 0$
$(a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2) = 0$ has four real roots.
$D_1 < 0$ and $D_2 < 0$
$(a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2) = 0$ has four complex roots.

5. If $D_1D_2 = 0$, then
Either $D_1 > 0$ and $D_2 = 0$ or $D_1 = 0$ and $D_2 > 0$
In this case $(a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2) = 0$
Has two equal real roots and two distinct real roots.
$D_1 < 0$ and $D_2 = 0$ or $D_1 = 0$ and $D_2 < 0$
In this case $(a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2)$
$= 0$ has two equal real roots and two imaginary roots.

Ex. Examine the characteristics of the roots for the equations $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$, where a, b, c, d are real numbers, and $ac = 2(b + d)$.

Sol. Consider D_1 and D_2 as the discriminants of the provided equations $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$, respectively.
Then $D_1 = a^2 - 4b$ and $D_2 = c^2 - 4d$.
Now
$$D_1 + D_2 = a^2 - 4b + c^2 - 4d$$
$$= a^2 + c^2 - 4(b + d)$$
$$= a^2 + c^2 - 2ac[\because ac = 2(b + d)]$$
$$= (a - c)^2 \geq 0$$
Consequently at least one of the given equations has real roots.