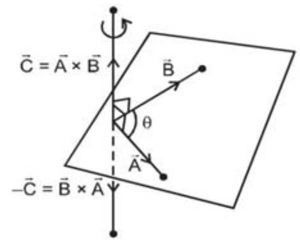
THE VECTOR PRODUCT



(A) If $\vec{a}$ & $\vec{b}$ are two vectors & θ is the angle between them, then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ such that $\vec{a}, \vec{b}$ & $\vec{n}$ forms a right handed screw system

Sign convention

(A) **Right handed screw system**

$\vec{a}, \vec{b}$ and $\hat{n}$ form a right handed system it means that if we rotate vector $\vec{a}$ towards the direction of $\vec{b}$. through the angle θ, then $\hat{n}$ advances in the same direction as a right handed screw would, if turned in the same way.

(B) **Lagranges Identity**

For any two vectors $\vec{a}$&$\vec{b}$; $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

(C) **Formulation of vector product in terms of scalar product**

The vector product $\vec{a} \times \vec{b}$ is the vector $\vec{c}$,, such that

A. $|\vec{c}| = \sqrt{\vec{a}^2\vec{b}^2 - (\vec{a} \cdot \vec{b})^2}$

B. $\vec{c} \cdot \vec{a} = 0; \vec{c} \cdot \vec{b} = 0$

C. $\vec{a}, \vec{b}, \vec{c}$ form a right handed system

(D) (i) $\vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a}$&$\vec{b}$ are parallel (collinear) $(\vec{a} \neq 0, \vec{b} \neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar

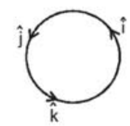
(ii) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (not commutative)

(iii) $(m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b}) = m(\vec{a} \times \vec{b})$ where m is a scalar.

(iv) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$ (distributive over addition)

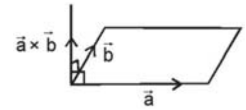
(v) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(vi) $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$



(E) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$&$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(F) Geometrically $|\vec{a} \times \vec{b}| =$ area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$.



(G) (i) Unit vector perpendicular to the plane of $\vec{a}$&$\vec{b}$ is $\hat{\mathbf{n}} = \pm\frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|}$

(ii) A vector of magnitude ' r ' & perpendicular to the plane of $\vec{a}$&$\vec{b}$ is $\pm\frac{\mathbf{r}(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$

(iii) If θ is the angle between $\vec{a}$&$\vec{b}$, then $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

(H) Vector area:

(i) If $\vec{a}, \vec{b}$&$\vec{c}$ are the pv's of 3 points A, B & C then the vector area of triangle

$$ABC = \frac{1}{2}\left[\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right]$$

(ii) The points A, B & C are collinear if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

(iii) Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$&$\vec{d}_2$ is given by $\frac{1}{2}|\vec{\mathbf{d}}_1 \times \vec{\mathbf{d}}_2|$.