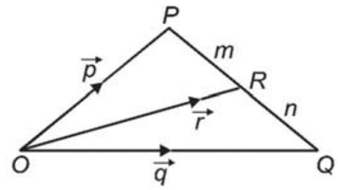
SECTION FORMULA

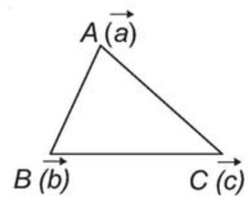
If R divides PQ in the ratio m: n, then



$$\vec{r} = \frac{m\vec{q} + n\vec{p}}{m + n}$$

If R be the mid-point of PQ then

$$\vec{r} = \frac{\vec{p} + \vec{q}}{2}$$



Again if G is the centroid of the triangle ABC, then

$$\vec{g} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

MAGNITUDE OF A VECTOR

Let $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ then $|\vec{A}| = \sqrt{a^2 + b^2 + c^2}$

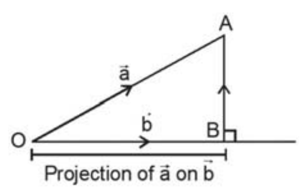
DOT PRODUCT (Scalar Product of two Vectors)

(A) $\vec{a} \cdot \vec{b} = \vec{a} \parallel \vec{b} \mid \cos\theta (0 \le \theta \le \pi)$, θ is angle between $\vec{a}$&$\vec{b}$.

Note that if θ is acute then $\vec{a} \cdot \vec{b} > 0$& if θ is obtuse then $\vec{a}\vec{b} < 0$

(B) $\vec{a} \cdot \vec{a} = |\vec{a}|^2 = \vec{a}^2, \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative)

$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ (distributive)



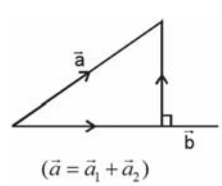
(C) $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}; (\vec{a}, \vec{b} \neq 0)$

(D) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1; \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

(E) Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

Note:

i. The vector component of $\vec{a}$ along $\vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{\vec{b}^2}\right)\vec{b}$ and perpendicular to $\vec{b} = \vec{a} - \left(\frac{\vec{a} \cdot \vec{b}}{\vec{b}^2}\right)\vec{b}$ [by triangle law of vector Addition]



ii. The angle ϕ between $\vec{a}$&$\vec{b}$ is given by $\cos\phi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ $0 \le \phi \le \pi$

iii. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$&$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$|\vec{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$, $|\vec{b}| = \sqrt{{b_1}^2 + {b_2}^2 + {b_3}^2}$

iv. $-|\vec{a}||\vec{b}| \le \vec{a}\cdot\vec{b} \le |\vec{a}||\vec{b}|$

v. Any vector $\vec{a}$ can be written as, $\vec{a} = (\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$

vi. A vector in the direction of the bisector of the angle between the two vectors

$\vec{a}$&$\vec{b}$ is $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$

Hence bisector of the angle between the two vectors $\vec{a}$&$\vec{b}$ is $\lambda(\hat{a} + \hat{b})$, where $\lambda \in R^+$.

Bisector of the exterior angle between $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} - \hat{b})$, $\lambda \in R^+$

vii. $|\vec{a} \pm \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2\vec{a}\cdot\vec{b}$

viii. $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}| + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$