

Chapter 13

Vectors

- Introduction
- Vectors and Scalars
- Types of Vectors
 - Zero Vector
 - Unit Vector
 - Co-initial Vectors
 - Equal Vectors
 - Negative of a Vector
 - Free Vectors
 - Localized Vectors
 - Parallel Vectors
 - Like and Unlike Vectors
 - Collinear Vectors
 - Non-collinear Vectors
 - Coplanar Vectors
- Scalar Multiplication
- Addition Composition
- Parallelogram Law of Addition of Vectors
- Linear Combination
- Expressions as Linear Combination
 - coplanar vectors
 - Arbitrary System of Vectors
- Linearly Dependent and Independent Vectors
- Section Formula
- Magnitude of a Vector
- Dot Product
- The Vector Product
- Scalar and Vector Triple Products and their Geometrical Interpretation
 - Scalar Triple Product
 - Vector Triple Product
 - Scalar Product Of Four Vectors
 - Vector Product Of Four Vectors
- Reciprocal System of Vectors
- Solving Vector Equation
- Applications to Geometry

INTRODUCTION

Vectors form a part of various mathematical systems that can be effectively utilized to address specific problems in Geometry, Mechanics, and other domains of Applied Mathematics. They enable the mathematical analysis of physical quantities that have both Magnitude and Direction, such as the velocity of a particle.

Physical quantities are broadly divided in two categories

(A) Vector Quantities (B) Scalar quantities.

(A) Vector Quantities

Any quantity, like velocity, momentum, or force, possessing both magnitude and direction, and for which vector addition holds significance, is categorized as a vector quantity. Quantities with magnitude and direction that do not adhere to the vector law of addition are not considered vectors.

For instance, the rotations of a rigid body involving finite angles possess both magnitude and direction, but they do not adhere to the vector addition law, making them non-vectors.

(B) Scalars Quantities

A scalar quantity, such as mass, length, time, density, or energy, is characterized by its size or magnitude without incorporating the notion of direction.

Mathematical Description Of Vector & Scalar

To gain a mathematical understanding of vectors, we will begin by comprehending directed line segments.

Directed line segment :

A Directed Line Segment is any specified section of a straight line, with identified initial and terminal points.

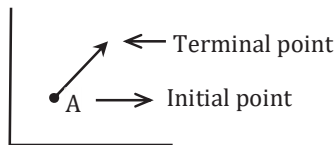
The directed line segment originating from point A and terminating at point B is represented by the symbol \overrightarrow{AB} .

The two endpoints of a directed line segment cannot be interchanged, and neither can the directed line segments.

\overrightarrow{AB} And \overrightarrow{BA} It is essential to consider them as distinct.

VECTORS AND SCALARS

Vectors

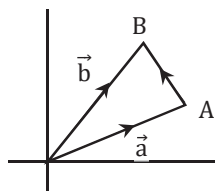


Vector quantities are characterized by both a specific magnitude and a specific direction. Typically, a vector is depicted by a directed line segment, denoted as \overrightarrow{AB} , where A is referred to as the initial point, and B is designated as the terminal point.

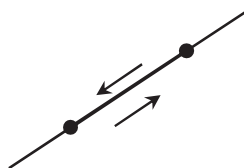
The magnitude of vector \overrightarrow{AB} is expressed by $|\overrightarrow{AB}|$.

Position Vector

Assume O is a fixed origin; the position vector of a point P is represented as \overrightarrow{OP} . If \overrightarrow{OP} denotes the position vectors of two points A and B, then the vector \overrightarrow{AB} can be expressed as $(\vec{B}) - (\vec{A})$, which is equivalent to the position vector of B minus the position vector of A.



A vector is defined as a directed line segment, and it possesses three fundamental characteristics.



(i) Length

The magnitude or length of the vector \overrightarrow{AB} is represented by the symbol $|\overrightarrow{AB}|$.

Clearly, we have $|\overrightarrow{AB}| = |\overrightarrow{BA}|$

(ii) Support

The infinite line that includes a directed line segment as a component is referred to as its line of support or simply the support.

(iii) Sense

The orientation of \overrightarrow{AB} is from point A to point B, while that of \overrightarrow{BA} is from B to A. Thus, the Direction of a directed line segment is determined from its initial point to its terminal point.

Scalar

Scalars pertain to quantities represented by a single real number used to measure magnitude (size). Measurements like voltage, mass, and temperature are examples of scalar quantities. Scalars are numerical values employed to gauge size, indicating the extent or scale of a particular property. Any real number can be considered a scalar, serving to express the magnitude of a quantity, such as 12.5 miles or 34° Celsius.