SCALAR MULTIPLICATION

If $\vec{a}$ is a vector and m is a scalar, then the product $m(\vec{a})$ results in a vector parallel to , with a magnitude equal to |m| times that of .

This operation is referred to as scalar multiplication. If $\vec{a}$ and $\vec{b}$ are vectors, and m and n are scalars, then the result is expressed as:

(i) $m(\vec{a}) = (\vec{a})m = m\vec{a}$

(ii) $m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$

(iii) $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

(iv) $m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

Note: In general, for any non-zero vectors, $\vec{a}, \vec{b} \& \vec{c}$ one may note that although $\vec{a}+\vec{b}+\vec{c}=0$ it is important to observe that while, it does not necessarily form the three sides of a triangle.

Ex. ABCD is a parallelogram with intersecting diagonals at P. If O is a constant point, then the expression is $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ equivalent to.

Sol. Since, P bisects both the diagonal AC and BD,

so $\overrightarrow{OA}+\overrightarrow{OC} = 2\overrightarrow{OP}$

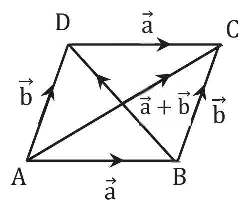
And $\overrightarrow{OB}+\overrightarrow{OD} = 2\overrightarrow{OP}$

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD} = 4\overrightarrow{OP}$

Ex. Given $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ that represent two adjacent sides of a parallelogram, determine unit vectors that are parallel to the diagonals of the parallelogram.

Sol. Let ABCD be a parallelogram such that

$\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$.



Then $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$

$\overrightarrow{AC} = \vec{a}+\vec{b} = 3\hat{i}+6\hat{j}-2\hat{k}$

$|\overrightarrow{AC}| = \sqrt{9+36+4} = 7$

$\overrightarrow{AB}+\overrightarrow{BD} = \overrightarrow{AD}$

$\overrightarrow{BD} = \overrightarrow{AD}-\overrightarrow{AB}$

$= \vec{b}-\vec{a} = \hat{i}+2\hat{j}-8\hat{k}$

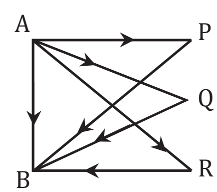
$|\overrightarrow{BD}| = \sqrt{1+4+64} = \sqrt{69}$

Unit vector along $\overrightarrow{AC} = \dfrac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \dfrac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$

Unit vector along $\overrightarrow{BD} = \dfrac{\overrightarrow{BD}}{|\overrightarrow{BD}|} = \dfrac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

Ex. A, B, P, Q, R are five points in a plane. If force $\overrightarrow{AP}, \overrightarrow{AQ}, \overrightarrow{AR}$ is applied to point A and force $\overrightarrow{PB}, \overrightarrow{QB}, \overrightarrow{RB}$ is applied to point B, then the resultant force is represented by.

Sol. From figure



$\overrightarrow{AP}+\overrightarrow{PB} = \overrightarrow{AB}$

$\overrightarrow{AQ}+\overrightarrow{QB} = \overrightarrow{AB}$

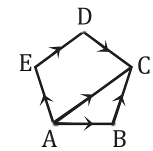
$\overrightarrow{AR}+\overrightarrow{RB} = \overrightarrow{AB}$

So, $(\overrightarrow{AP}+\overrightarrow{AQ}+\overrightarrow{AR})+(\overrightarrow{PB}+\overrightarrow{QB}+\overrightarrow{RB}) = 3\overrightarrow{AB}$

So, required resultant $= 3\overrightarrow{AB}$

Ex. ABCDE is a pentagon. Demonstrate that the resultant of the forces $\overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{ED}$ and $\overrightarrow{AC}$ is $3\overrightarrow{AC}$. is.

Sol. Let $\vec{R}$ be the resultant force



$$\vec{R} = \overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$$

$$\vec{R} = (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{AE} + \overline{ED} + \overrightarrow{DC}) + \overrightarrow{AC}$$

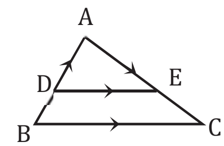
$$\vec{R} = \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$$

$$R = 3\overrightarrow{AC}$$ Hence proved

Ex. Demonstrate that the line connecting the midpoints of two sides of a triangle is parallel to the third side and is half of its length.

Sol. Consider the midpoints of side AB and AC of triangle ABC as D and E, respectively.

Now in ΔABC, by triangle law of addition



$$\overrightarrow{BA} = 2\overrightarrow{DA}$$

$$\overrightarrow{AC} = 2\overrightarrow{AE}$$

Now, in ΔABC,, applying the triangle law of addition

$$\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$$

$$2\overrightarrow{DA} + 2\overrightarrow{AE} = \overrightarrow{BC}$$

$$\overrightarrow{DA} + \overrightarrow{AE} = \frac{1}{2}\overrightarrow{BC}$$

$$\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$$

Therefore, the line DE is parallel to the third side BC of the triangle and is half of its length.