

PARALLELOGRAM LAW OF ADDITION OF VECTORS

If two vectors are represented in magnitude and direction by the two consecutive sides of a parallelogram, then their sum will be represented by the diagonal passing through the co-initial point.

Consider vectors \vec{a} and \vec{b} originating from point O, represented by line segments. Now, complete the parallelogram OPRQ.

The vector represented by the diagonal OR will denote the sum of the vectors. .

$$\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR}$$

$$\vec{a} + \vec{b} = \vec{OR}$$

The process of adding two vectors using this technique is known as the Parallelogram Law of Vector Addition.

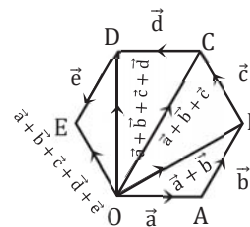
Properties of vector addition

1. If two vectors \vec{a} and \vec{b} are represented by \overrightarrow{OA} and \overrightarrow{OB} , then their sum $\vec{a} + \vec{b}$ is a vector represented by \overrightarrow{OC} , where OC is the diagonal of the parallelogram OACB.
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (Commutative)
3. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (Associative)
4. $\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$ (Additive identity)
5. $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$ (Additive inverse)
6. $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
7. $|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$

Polygon law of vector Addition (Addition of more than two vectors)

The addition of more than two vectors is accomplished through the repetition of the Triangle Law.

Suppose we need to determine the sum of five vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and \vec{e} . If these vectors are represented by line segments forming a polygon $\overrightarrow{OA}, \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ and \overrightarrow{DE} respectively, their sum will be denoted by \overrightarrow{OE} . This vector is represented by the remaining (last) side of the polygon OABCDE in reverse order. This can also be clarified as follows:



By triangle's law

$$\begin{array}{ll} \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} & \text{Or} \quad \vec{a} + \vec{b} = \overrightarrow{OB} \\ \overrightarrow{OB} + \vec{c} = \overrightarrow{OC} & \text{Or} \quad (\vec{a} + \vec{b}) + \vec{c} = \overrightarrow{OC} \\ \overrightarrow{OC} + \vec{d} = \overrightarrow{OD} & \text{Or} \quad (\vec{a} + \vec{b} + \vec{c}) + \vec{d} = \overrightarrow{OD} \\ \overrightarrow{OD} + \vec{e} = \overrightarrow{OE} & \text{Or} \quad (\vec{a} + \vec{b} + \vec{c} + \vec{d}) + \vec{e} = \overrightarrow{OE} \end{array}$$

In this context, \overrightarrow{OE} is depicted as the line segment connecting the initial point o of the first vector \vec{a} and the terminal point of the last vector \vec{e} . To determine the sum of more than two vectors using this approach, a polygon is constructed. Hence, this technique is referred to as the polygon law of addition. If the initial point of the first vector and the terminal point of the last vector coincide, the sum of the vectors will result in a null vector.