LINEARLY DEPENDENT AND INDEPENDENT VECTORS

Let $x_1, x_2 \ldots x_n$ are scalar and $\vec{a}_1, \vec{a}_2 \ldots \vec{a}_n$ are n vectors such that $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 + \ldots x_n\vec{a}_n = 0$

Here find $x_1, x_2, x_3, \ldots . x_n$ by comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$.

If all $x_1 = x_2 = x_3 = \cdots .. = x_n = 0$, then vectors are linearly independent and if at least one of $x_1, x_2, \ldots \ldots x_n$ is non-zero, then vectors are linearly dependent. Consequently, therefore

i. Two parallel vectors are always linearly dependent and two non - parallel are always linearly independent. If $\vec{a}$ and $\vec{b}$ are two parallel vectors, then we write always $\vec{a} = t\vec{b}$.

ii. Two non-parallel vectors are always linearly independent.

iii. If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then they will be dependent and in this case we may $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where x, y, z are scalars and all three are not zero together.

$x\vec{a} + y\vec{b} + z\vec{c} = 0$, where x, y, z are scalars and all three are not zero together.

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent

$\vec{c} = a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$

then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

iv. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and the vectors $x_1\vec{a} + y_1\vec{b} + z_1\vec{c}, x_2\vec{a} + y_2\vec{b} + z_2\vec{c}, x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$ are coplanar, then

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

v. Any four vectors are always linearly dependent.