ORTHOGONAL TRAJECTORY

Let a family of curves be represented by the equation

$$f(x, y. c) = 0 \quad \ldots(1)$$

And we want to determine the family of curves each member of which cuts members of the given family (i) at right angles. Then the family of curves we are seeking constitute, the orthogonal trajectory of family (i).

The procedure can be systematically described as thus:

(i) We first form the differential equation of the family by differentiating (i) and eliminating the arbitrary constants.

(ii) As products of slopes of tangent at a point of intersection of (i) and its orthogonal trajectory is -1, we replace $\frac{dy}{dx}$ with $-\frac{dx}{dy}$ in the differential equation of family (i).

(iii) Now we solve the resulting differential equation to obtain the family of orthogonal trajectory.