Chapter 11

Differential Equations

INTRODUCTION

An equation that involves independent and dependent variables along with the derivatives of the dependent variables is referred to as a differential equation. There are two main types of differential equations:

Ordinary Differential Equation:

If the dependent variables are contingent on a single independent variable, denoted as x, then the differential equation is classified as ordinary.

For example

$$\frac{dy}{dx}+\frac{dz}{dx}=y+z$$

$$\frac{dy}{dx}+xy=\sin x$$

$$\frac{d^3y}{dx^3}+2\frac{dy}{dx}+y=e^x$$

$$k\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$$

$$y=x\frac{dy}{dx}+k\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}$$

Partial differential equation

If the dependent variables rely on two or more independent variables, it is termed as a partial differential equation.

For example $\quad y^2\frac{\partial z}{\partial x}+y\frac{\partial^2 z}{\partial y}=ax,\quad \frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$

ORDER AND DEGREE OF A DIFFERENTIAL EQUATIONS

Order of Differential Equation

The order of a differential equation is determined by the highest order of the differential coefficient present in it.

Degree of Differential Equation

The order of a differential equation is established by the highest degree of the leading derivative once the differential equation is simplified by removing radicals and fractions related to the derivatives.

Thus the differential equation: $f(x,y)\left[\frac{d^m y}{dx^m}\right]+\phi(x,y)\left[\frac{d^{m-1}(y)}{dx^{m-1}}\right]^q+........=0$ is of order m & degree p.

Remember

The exponents of all the differential coefficients must be devoid of radicals and fractions. Additionally, the degree is always a positive natural number, and it may or may not exist for a given differential equation.

Ex. Determine the order and degree of the following differential equations.

(i) $\frac{d^2y}{dx^2}=\left[y+\left(\frac{dy}{dx}\right)^6\right]^{1/4}$ (ii) $y=e^{\left(\frac{dy}{dx}\frac{d^2y}{dx^2}\right)}$

(iii) $\sin\left(\frac{dy}{dx}+\frac{d^2y}{dx^2}\right)=y$ (iv) $e^{y''}-xy''+y=0$

Sol. (i) $\left(\frac{d^2y}{dx^2}\right)^4=y+\left(\frac{dy}{dx}\right)^6$

order $=2$, degree $=4$

(ii) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=\ln y$

order $=2$, degree $=1$

(iii) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=\sin^{-1}y$

order $=2$, degree $=1$

(iv) $e^{\frac{d^2y}{dx^3}}-x\frac{d^2y}{dx^2}+y=0$

The equation cannot be formulated as a polynomial in the differential coefficients, making the concept of degree inapplicable. However, the order is 3.