FORMATION OF AN ORDINARY DIFFERENTIAL EQUATIONS (ODE)

To derive a differential equation with a solution that

$$f(x_1, y_1, c_1, c_2, c_3.........,c_n) = 0$$

Involving c_1, c_2, c_n, which are 'n' arbitrary constants, we need to eliminate these 'n' constants, and for this purpose, we require (n+1) equations.

A differential equation is derived in the following manner:

A. Take the given equation and differentiate it with respect to the independent variable (let's say x) as many times as there are independent arbitrary constants in the equation.
B. Remove the arbitrary constants.
C. The resulting eliminate represents the desired differential equation.

A differential equation depicts a collection of curves, each adhering to shared properties. This can be viewed as the geometric interpretation of the differential equation.

For n differentiations, the resulting equation must include a derivative of nth order, i.e., equal to the number of independent arbitrary constants.

Ex. Derive a differential equation for the family of straight lines that pass through the origin.

Sol. The family of straight lines passing through the origin is represented by the equation y = mx, where 'm' is a parameter. Upon differentiation with respect to x,

$$\frac{dy}{dx} = m$$

By removing 'm' from both equations, we acquire

$$\frac{dy}{dx} = \frac{y}{x}$$

Which is the required differential equation.

Ex. Determine the differential equation for all parabolas with axes parallel to the x-axis and a given latus rectum of length 'a'.

Sol. The equation of a parabola with an axis parallel to the x-axis and a latus rectum of length 'a' is

$$(y - \beta)^2 = a\ (x - a)$$

Upon differentiation of both sides, we obtain:

$$2(y - \beta)\frac{dy}{dx} = a$$

Upon further differentiation,

$$2(y - \beta)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

$$a\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^3 = 0$$