17. $\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+C$

18. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x+\sqrt{x^2+a^2}|+C$

OR

$$\sinh^{-1}\frac{x}{a}+C$$

19. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x+\sqrt{x^2-a^2}|+C$

OR

$$\cos^{-1}\frac{x}{a}+C$$

20. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$

21. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$

22. $\int \sqrt{a^2-x^2}dx = \frac{a^2}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-x}+C$

23. $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|+C$

24. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|+C$

25. $\int e^{ax}\cdot\sin bx dx = \frac{e^{ax}}{a^2+h^2}(a\sin bx - b\cos bx)+c$

26. $\int e^{ax}\cdot\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$

Theorems on integration:

1. $\int Cf(x)\cdot dx = C\int f(x)dx$

2. $\int[f(x)\pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

3. $\int f(x)dx = g(x)+C_1$

$$\int f(ax+b)dx = \frac{g(ax+b)}{a}+C_2$$

Ex. Solve: $\int 4x^5dx$

Sol. $\int 4x^5dx$

$$\frac{4}{6}x^6+C$$

$$\frac{2}{3}x^6+C$$

Ex. Evaluate: $\int\left(x^3+5x^2-4+\frac{7}{x}+\frac{2}{\sqrt{x}}\right)dx$

Sol. $\int\left(x^3+5x^2-4+\frac{7}{x}+\frac{2}{\sqrt{x}}\right)dx$

$$=\int x^3dx+\int 5x^2dx-\int 4dx+\int\frac{7}{x}dx+\int\frac{2}{\sqrt{x}}dx$$

$$=\int x^3dx+5.\int x^2dx-4.\int 1dx+7.\int\frac{1}{x}dx+2\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^4}{4}+5.\frac{x^3}{3}-4x+7\text{In}|x|+2\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)+C$$

$$=\frac{x^4}{4}+\frac{5}{3}x^3-4x+7\text{In}|x|+4\sqrt{x}+C$$

Ex. Evaluate: $\int\left(e^{2\ell nx}+e^{a\ell nx}+e^{4\ell nx}\right)dx, a>0$

Sol. $\int\left(e^{2\ell nx}+e^{a\ell nx}+e^{4\ell nx}\right)dx$

$$=\int\left(e^{\ell nx^2}+e^{\ell nx^a}+e^{\ell nx^4}\right)dx$$

$$=\int\left(x^2+x^3+x^4\right)dx$$

$$=\frac{x^3}{3}+\frac{x^{3+1}}{a+1}+\frac{x^5}{5}+c$$

Ex. Evaluate: $\int\left(\frac{2^{x+1}-5^{x-1}}{10^x}\right)dx$

Sol. $\int\left(\frac{2^{x+1}-5^{x-1}}{10^x}\right)dx$

$$=\int\left[2\left(\frac{1}{5}\right)^x-\frac{1}{5}\left(\frac{1}{2}\right)^x\right]dx$$

$$=\frac{2\left(\frac{1}{5}\right)^x}{\log_e\left(\frac{1}{5}\right)}-\frac{1}{5}\frac{\left(\frac{1}{2}\right)^x}{\log\left(\frac{1}{2}\right)}+C$$

Ex. Evaluate: $\int\sec^2 x\,\text{cosec}^2 x\,dx$

Sol. $I=\int\sec^2 x\,\text{cosec}^2 x$

$$\int\frac{\cos^2 x+\sin^2 x}{\cos^2 x\sin^2 x}$$

$$\int\left(\sec^2 x+\text{cosec}^2 x\right)dx$$

$$\tan x-\cot x+C$$

Ex. Evaluate: $\int \frac{(1+x)^3}{\sqrt{x}}\,dx$

Sol. $\int \frac{(1+x)^3}{\sqrt{x}}dx$

$$\int \frac{1+3x+3x^2+x^3}{\sqrt{x}}dx$$

$$\int x^{-\frac{1}{2}} + 3\int x^{\frac{1}{2}}dx + 3\int x^{\frac{3}{2}}dx + \int x^{\frac{5}{2}}dx$$

$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C$$

$$2\sqrt{x} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}} + C$$

Ex. Evaluate: $\int \frac{1}{4+9x^2}\,dx$

Sol. We have $\int \frac{1}{4+9x^2}dx$

$$\frac{1}{9}\int \frac{1}{\frac{4}{9}+x^2}dx$$

$$\frac{1}{9}\int \frac{1}{\left(\frac{2}{3}\right)^2+x^2}dx$$

$$\frac{1}{9}\cdot\frac{1}{\left(\frac{2}{3}\right)}\tan^{-1}\frac{x}{\left(\frac{2}{3}\right)} + C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + C$$

Ex. Evaluate: $\int \cos x \cos 2x\,dx$

Sol. $\int \cos x\cos 2x dx$

$$\frac{1}{2}\int 2\cos x\cos 2x dx$$

$$\frac{1}{2}\int(\cos 3x + \cos x)dx$$

$$\frac{1}{2}\left(\frac{\sin 3x}{3} + \sin x\right) + C$$