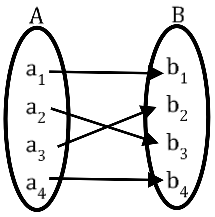
TYPE OF FUNCTION

One to One Function

A function f: A → B is considered one-to-one if each element in A corresponds to a distinct element in B. This property is also known as injective.

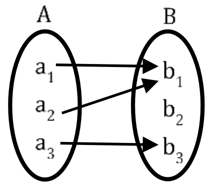
if $a_1 \in A$ and $a_2 \in B$, f is defined as f: A → B such that

f (a_1) = f (a_2).



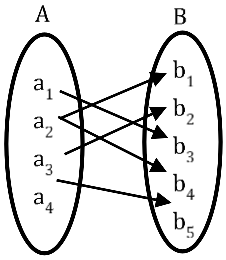
Many to one function

It refers to a function that maps two or more elements of A to a common element in set B. Multiple elements in A share the same image in B.



Onto Function

If a function exists where each element in set B has at least one pre-image in set A, it is referred to as an Onto Function. Onto functions are also known as Surjective Functions.



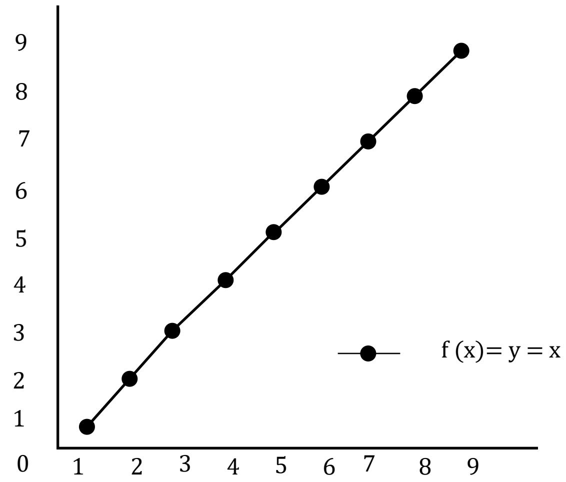
DIFFERENT TYPES OF FUNCTIONS

Other Types of Functions

A function is uniquely characterized by its graph, which essentially consists of a set of coordinate pairs representing x and f(x). Let's delve into a deeper understanding of the various types of functions and their corresponding graphs.

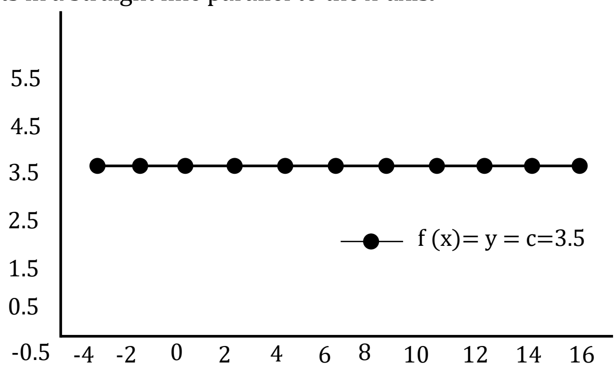
Identity Function

Consider the set of real numbers, denoted as R. If the function $f : R \rightarrow R$ is defined as $f(x) = y = x$, where $x \in R$, then this function is recognized as the Identity function. The domain and range of the function encompass all real numbers (R). The graph of the Identity function is consistently a straight line and intersects the origin.



Constant Function

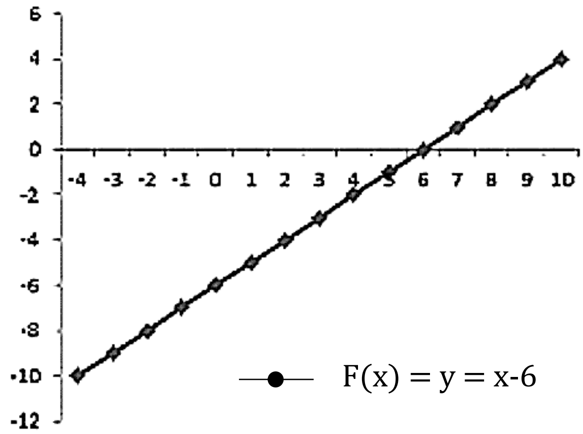
If the function $f : R \rightarrow R$ is defined as $f(x)=y=c$, where $x \in R$ and c is a constant in R, then this function is referred to as a Constant function. The domain of the function f is R, and its range is a constant, c. When graphed, it results in a straight line parallel to the x-axis.



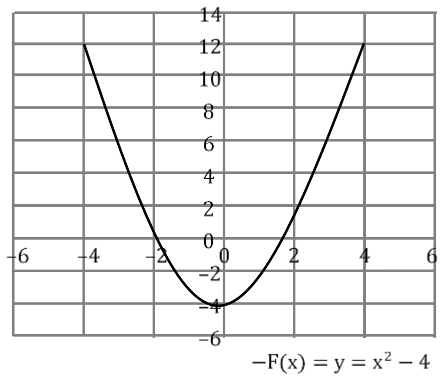
Polynomial Function

A polynomial function is expressed as $y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where n is a non-negative integer, and $a_0, a_1, a_2, \dots\dots, a_n \in R$. The degree of the polynomial function is determined by the highest power in the expression. Polynomial functions are categorized based on their degrees:

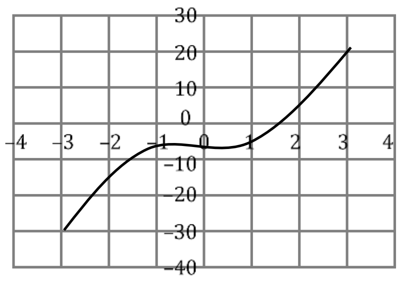
- Constant Function: If the degree is zero, the polynomial function becomes a constant function, as explained above.
- Linear Function: A polynomial function with a degree of one, represented by expressions such as $y = x + 1$ or $y = x$ or $y = 2x - 5$. where both the domain and range are R. The graph of a linear function is invariably a straight line.



- Quadratic Function: When the degree of a polynomial function is two, it becomes a quadratic function. The expression for a quadratic function is given by $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a,b,c are constants, and x is a variable. Both the domain and range of a quadratic function are R. An example of a quadratic function is $f(x) = x^2 - 4$. The graphical representation of such a quadratic function is characterized by a parabolic shape

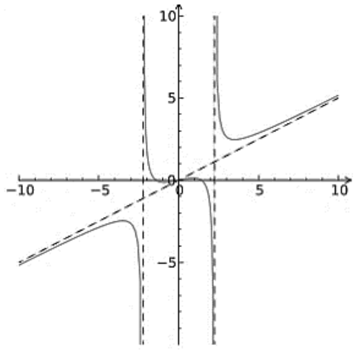


- Cubic Function: A cubic polynomial function, characterized by a degree of three, is represented as $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, and d are constants, with x as the variable. An example of a cubic function is $f(x) = y = x^3 - 5$. The domain and range of a cubic function are both R. The graphical representation of such a cubic function typically exhibits a distinctive shape.



Rational Function

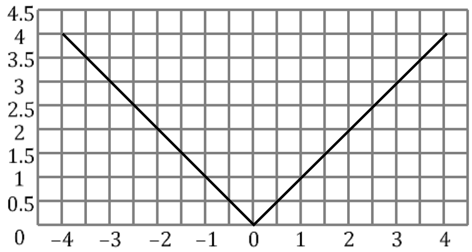
A rational function is defined as any function that can be expressed as a rational fraction, denoted as $\frac{f(x)}{g(x)}$ where both the numerator f(x) and denominator g(x) are polynomial functions of x, and $g(x) \neq 0$. Consider a function f: R → R defined as $f(x) = \frac{1}{x+2.5}$. The domain and range of this function are R. The graphical representation of a rational function often exhibits asymptotes, curves that appear to touch the axes lines.



Modulus Function

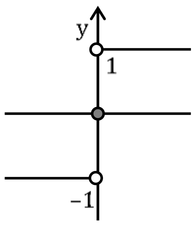
The absolute value of a number c is denoted as |c|. If we consider a function f: R→ R defined as f(x) = |x|, it is referred to as the Modulus Function. For each non–negative value of x, f(x) = x, and for each negative value of x, f(x) = -x, meaning

f (x) = {x, if x ≥ 0; – x, if x < 0. The graph of this function, shown below, demonstrates that both the domain and range span R.



Signum Function

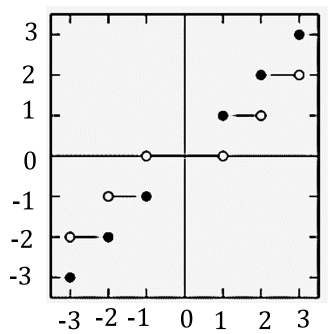
A function f: R→ R is given by f(x) = { 1, if x > 0; 0, if x = 0; -1, if x < 0 . This function, commonly known as the Signum or sign function, assigns values based on the sign of the real number x.



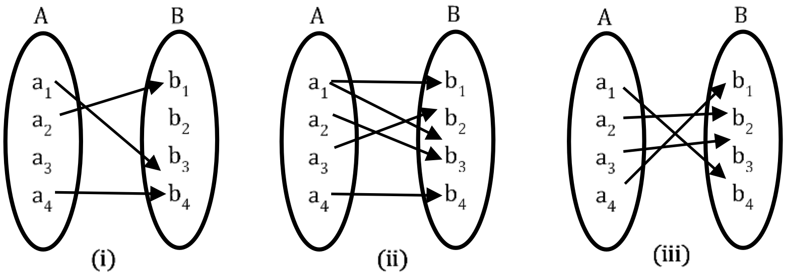
Greatest Integer Function

If a function f : **R→ R is expressed as** f(x) = [x], where x belongs to the set X, it rounds off the real number to the integer less than or equal to that number. For example, within the interval (k, k+1), the value of the greatest integer function is k, where k is an integer.

For example: [-21] = 21, [5.12] = 5. The graphical representation is



Ex. Which one of the following represents a function?



Sol. Figure (iii) exemplifies a function as it systematically maps each element of set A to a corresponding element in set B. On the contrary, in figure (ii), the function assigns one element of A to two elements in B, illustrating a one-to-many relationship. In figure (i), the given function violates the definition of a function, as it does not map every element of A to an element in B.

Ex. What does the term "function" mean, and what are its various types?

Sol. A function is a unique relation that associates each element of one set with precisely one element from another set. The different types of functions include:

- Many to one function
- One to one function
- Onto function
- One and onto function
- Constant function
- Identity function
- Quadratic function
- Polynomial function
- Modulus function
- Rational function
- Signum function
- Greatest integer function

DIFFERENT TYPES OF FUNCTIONS

Algebra of Real Functions

We will explore the addition, subtraction, multiplication, and division of real mathematical functions together.

Addition of Two Real Functions

Consider two real-valued functions, f and g, where f: X→R and g: X→R, with X ⊂ R. The addition of these functions, denoted as (f + g): X→R, is defined as follows:

$$(f+g)(x) = f(x) + g(x), \text{ for all } x \in X$$

Subtraction of One Real Function from the Other

Consider two real functions, f and g, where f: X→R and g: X→R with X ⊂ R.
Let f: X→R and g: X→R be two real functions where X ⊂ R. The subtraction of these functions denoted as (f – g): X→R is defined by:

$$(f-g)(x) = f(x) - g(x), \text{ for all } x \in X.$$

Multiplication by a Scalar

Consider a real–valued function f: X→R, and let γ be any scalar (real number). The product of the real number f by the scalar γ, denoted as (γf): X→R, is defined as follows:

$$(\gamma f)(x) = \gamma f(x), \text{ for all } x \in X$$

Multiplication of Two Real Functions

The product of two real functions say, denoted as f and g where f: X→R and g: X→R, is defined as:

$$(fg)(x) = f(x)g(x), \text{ for all } x \in X.$$

Division of Two Real Functions

Let f and g be two real-valued functions such that f: X→R and g: X→R where X ⊂ R. The quotient of these two functions $\left(\frac{f}{g}\right)$: X→R is defined by:

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} \text{ for all } x \in X$$

Note: It is alternatively known as element-wise multiplication.

Ex. Let $f(x) = x^3$ and $g(x) = 3x + 1$ and a scalar, γ= 6. Find

1. $(f+g)(x)$
2. $(f-g)(x)$
3. $(\gamma f)(x)$
4. $(\gamma g)(x)$
5. $(fg)(x)$
6. $(f/g)(x)$

Sol. We have

1. $(f+g)(x) = f(x) + g(x) = x^3 + 3x + 1$
2. $(f-g)(x) = f(x) - g(x) = x^3 - (3x+1) = x^3 - 3x - 1$
3. $(\gamma f)(x) = \gamma f(x) = 6x^3$
4. $(\gamma g)(x) = \gamma g(x) = 6(3x+1) = 18x + 6.$
5. $(fg)(x) = f(x)g(x) = x^3(3x+1) = 3x^4 + x^3.$
6. $(f/g)(x) = f(x)/g(x) = x^3/(3x+1)$, Provided $x \neq -1/3$